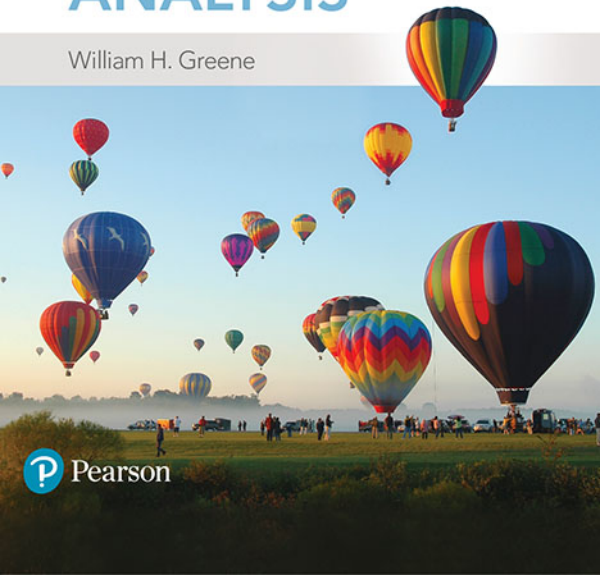


EIGHTH EDITION

# ECONOMETRIC ANALYSIS

William H. Greene



 Pearson

Percentiles of the Chi-Squared Distribution. Table Entry Is  $c$  Such That  
 $\text{Prob}[\chi_n^2 \leq c] = P$

$n$	.005	.010	.025	.050	.100	.250	.500	.750	.900	.950	.975	.990	.995
1	.00004	.0002	.001	.004	.02	.10	.45	1.32	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	.58	1.39	2.77	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	1.21	2.37	4.11	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	1.92	3.36	5.39	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.70	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	15.98	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	16.34	20.34	24.93	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	17.24	21.34	26.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	18.14	22.34	27.14	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	19.04	23.34	28.24	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	19.94	24.34	29.34	34.38	37.65	40.65	44.31	46.93
30	13.79	14.95	16.79	18.49	20.60	24.48	29.34	34.80	40.26	43.77	46.98	50.89	53.67
35	17.19	18.51	20.57	22.47	24.80	29.05	34.34	40.22	46.06	49.80	53.20	57.34	60.27
40	20.71	22.16	24.43	26.51	29.05	33.66	39.34	45.62	51.81	55.76	59.34	63.69	66.77
45	24.31	25.90	28.37	30.61	33.35	38.29	44.34	50.98	57.51	61.66	65.41	69.96	73.17
50	27.99	29.71	32.36	34.76	37.69	42.94	49.33	56.33	63.17	67.50	71.42	76.15	79.49

EIGHTH EDITION  
ECONOMETRIC ANALYSIS



**William H. Greene**

*The Stern School of Business*

*New York University*



New York, NY

For Margaret and Richard Greene

Vice President, Business Publishing: Donna Battista  
Director of Portfolio Management: Adrienne D' Ambrosio  
Director, Courseware Portfolio Management: Ashley Dodge  
Senior Sponsoring Editor: Neeraj Bhalla  
Editorial Assistant: Courtney Paganelli  
Vice President, Product Marketing: Roxanne McCarley  
Director of Strategic Marketing: Brad Parkins  
Strategic Marketing Manager: Deborah Strickland  
Product Marketer: Tricia Murphy  
Field Marketing Manager: Ramona Elmer  
Product Marketing Assistant: Jessica Quazza  
Vice President, Production and Digital Studio, Arts and Business: Etain O'Dea  
Director of Production, Business: Jeff Holcomb  
Managing Producer, Business: Alison Kalil  
Content Producer: Sugandh Juneja  
Operations Specialist: Carol Melville

Creative Director: Blair Brown  
Manager, Learning Tools: Brian Surette  
Content Developer, Learning Tools: Lindsey Sloan  
Managing Producer, Digital Studio, Arts and Business: Diane Lombardo  
Digital Studio Producer: Melissa Honig  
Digital Studio Producer: Alana Coles  
Digital Content Team Lead: Noel Lotz  
Digital Content Project Lead: Courtney Kamauf  
Full-Service Project Management and Composition: SPi Global  
Interior Design: SPi Global  
Cover Design: SPi Global  
Cover Art: Jim Lozouski/Shutterstock  
Printer/Binder: RRD Crawfordsville  
Cover Printer: Phoenix/Hagerstown

Microsoft and/or its respective suppliers make no representations about the suitability of the information contained in the documents and related graphics published as part of the services for any purpose. All such documents and related graphics are provided "as is" without warranty of any kind. Microsoft and/or its respective suppliers hereby disclaim all warranties and conditions with regard to this information, including all warranties and conditions of merchantability, whether express, implied or statutory, fitness for a particular purpose, title and non-infringement. In no event shall Microsoft and/or its respective suppliers be liable for any special, indirect or consequential damages or any damages whatsoever resulting from loss of use, data or profits, whether in an action of contract, negligence or other tortious action, arising out of or in connection with the use or performance of information available from the services.

The documents and related graphics contained herein could include technical inaccuracies or typographical errors. Changes are periodically added to the information herein. Microsoft and/or its respective suppliers may make improvements and/or changes in the product(s) and/or the program(s) described herein at any time. Partial screen shots may be viewed in full within the software version specified.

Microsoft® and Windows® are registered trademarks of the Microsoft Corporation in the U.S.A. and other countries. This book is not sponsored or endorsed by or affiliated with the Microsoft Corporation.

Copyright © 2018, 2012, 2008 by Pearson Education, Inc. or its affiliates. All Rights Reserved. Manufactured in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms, and the appropriate contacts within the Pearson Education Global Rights and Permissions department, please visit [www.pearsoned.com/permissions/](http://www.pearsoned.com/permissions/).

Acknowledgments of third-party content appear on the appropriate page within the text.

PEARSON and ALWAYS LEARNING are exclusive trademarks owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks, logos, or icons that may appear in this work are the property of their respective owners, and any references to third-party trademarks, logos, icons, or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc., or its affiliates, authors, licensees, or distributors.

#### Library of Congress Cataloging-in-Publication Data on File

## BRIEF CONTENTS



Examples and Applications

Preface

### **Part I      The Linear Regression Model**

Chapter 1	Econometrics	1
Chapter 2	The Linear Regression Model	12
Chapter 3	Least Squares Regression	28
Chapter 4	Estimating the Regression Model by Least Squares	54
Chapter 5	Hypothesis Tests and Model Selection	113
Chapter 6	Functional Form, Difference in Differences, and Structural Change	153
Chapter 7	Nonlinear, Semiparametric, and Nonparametric Regression Models	202
Chapter 8	Endogeneity and Instrumental Variable Estimation	242

### **Part II      Generalized Regression Model and Equation Systems**

Chapter 9	The Generalized Regression Model and Heteroscedasticity	297
Chapter 10	Systems of Regression Equations	326
Chapter 11	Models for Panel Data	373

### **Part III      Estimation Methodology**

Chapter 12	Estimation Frameworks in Econometrics	465
Chapter 13	Minimum Distance Estimation and the Generalized Method of Moments	488
Chapter 14	Maximum Likelihood Estimation	537
Chapter 15	Simulation-Based Estimation and Inference and Random Parameter Models	641
Chapter 16	Bayesian Estimation and Inference	694

### **Part IV      Cross Sections, Panel Data, and Microeconometrics**

Chapter 17	Binary Outcomes and Discrete Choices	725
------------	--------------------------------------	-----

**iv**      **Brief Contents**

Chapter 18	Multinomial Choices and Event Counts	826
Chapter 19	Limited Dependent Variables—Truncation, Censoring, and Sample Selection	918

**Part V      Time Series and Macroeconometrics**

Chapter 20	Serial Correlation	981
Chapter 21	Nonstationary Data	1022
References		1054

Index		1098
-------	--	------

**Part VI      Online Appendices**

Appendix A	Matrix Algebra	A-1
Appendix B	Probability and Distribution Theory	B-1
Appendix C	Estimation and Inference	C-1
Appendix D	Large-Sample Distribution Theory	D-1
Appendix E	Computation and Optimization	E-1
Appendix F	Data Sets Used in Applications	F-1

## Contents



**Examples and Applications**    xxiv

**Preface**    xxxv

### **Part I The Linear Regression Model**

#### **CHAPTER 1 Econometrics**    1

- 1.1 Introduction    1
- 1.2 The Paradigm of Econometrics    1
- 1.3 The Practice of Econometrics    3
- 1.4 Microeconometrics and Macroeconometrics    4
- 1.5 Econometric Modeling    5
- 1.6 Plan of the Book    8
- 1.7 Preliminaries    9
  - 1.71 Numerical Examples*    9
  - 1.72 Software and Replication*    10
  - 1.73 Notational Conventions*    10

#### **CHAPTER 2 The Linear Regression Model**    12

- 2.1 Introduction    12
- 2.2 The Linear Regression Model    13
- 2.3 Assumptions of the Linear Regression Model    16
  - 2.3.1 Linearity of the Regression Model*    17
  - 2.3.2 Full Rank*    20
  - 2.3.3 Regression*    22
  - 2.3.4 Homoscedastic and Nonautocorrelated Disturbances*    23
  - 2.3.5 Data Generating Process for the Regressors*    25
  - 2.3.6 Normality*    25
  - 2.3.7 Independence and Exogeneity*    26
- 2.4 Summary and Conclusions    27

#### **CHAPTER 3 Least Squares Regression**    28

- 3.1 Introduction    28
- 3.2 Least Squares Regression    28

3.2.1	<i>The Least Squares Coefficient Vector</i>	29
3.2.2	<i>Application: An Investment Equation</i>	30
3.2.3	<i>Algebraic Aspects of the Least Squares Solution</i>	33
3.2.4	<i>Projection</i>	33
3.3	Partitioned Regression and Partial Regression	35
3.4	Partial Regression and Partial Correlation Coefficients	38
3.5	Goodness of Fit and the Analysis of Variance	41
3.5.1	<i>The Adjusted R-Squared and a Measure of Fit</i>	44
3.5.2	<i>R-Squared and the Constant Term in the Model</i>	47
3.5.3	<i>Comparing Models</i>	48
3.6	Linearly Transformed Regression	48
3.7	Summary and Conclusions	49
<b>CHAPTER 4</b>	<b>Estimating the Regression Model by Least Squares</b>	<b>54</b>
4.1	Introduction	54
4.2	Motivating Least Squares	55
4.2.1	<i>Population Orthogonality Conditions</i>	55
4.2.2	<i>Minimum Mean Squared Error Predictor</i>	56
4.2.3	<i>Minimum Variance Linear Unbiased Estimation</i>	57
4.3	Statistical Properties of the Least Squares Estimator	57
4.3.1	<i>Unbiased Estimation</i>	59
4.3.2	<i>Omitted Variable Bias</i>	59
4.3.3	<i>Inclusion of Irrelevant Variables</i>	61
4.3.4	<i>Variance of the Least Squares Estimator</i>	61
4.3.5	<i>The Gauss–Markov Theorem</i>	62
4.3.6	<i>The Normality Assumption</i>	63
4.4	Asymptotic Properties of the Least Squares Estimator	63
4.4.1	<i>Consistency of the Least Squares Estimator of <math>\beta</math></i>	63
4.4.2	<i>The Estimator of Asy. Var[b]</i>	65
4.4.3	<i>Asymptotic Normality of the Least Squares Estimator</i>	66
4.4.4	<i>Asymptotic Efficiency</i>	67
4.4.5	<i>Linear Projections</i>	70
4.5	Robust Estimation and Inference	73
4.5.1	<i>Consistency of the Least Squares Estimator</i>	74
4.5.2	<i>A Heteroscedasticity Robust Covariance Matrix for Least Squares</i>	74
4.5.3	<i>Robustness to Clustering</i>	75
4.5.4	<i>Bootstrapped Standard Errors with Clustered Data</i>	77
4.6	Asymptotic Distribution of a Function of $\mathbf{b}$ : The Delta Method	78
4.7	Interval Estimation	81
4.7.1	<i>Forming a Confidence Interval for a Coefficient</i>	81
4.7.2	<i>Confidence Interval for a Linear Combination of Coefficients: the Oaxaca Decomposition</i>	83



4.8	Prediction and Forecasting	86	
	4.8.1	<i>Prediction Intervals</i>	86
	4.8.2	<i>Predicting <math>y</math> when the Regression Model Describes <math>\log y</math></i>	87
	4.8.3	<i>Prediction Interval for <math>y</math> when the Regression Model Describes <math>\log y</math></i>	88
	4.8.4	<i>Forecasting</i>	92
4.9	Data Problems	93	
	4.9.1	<i>Multicollinearity</i>	94
	4.9.2	<i>Principal Components</i>	97
	4.9.3	<i>Missing Values and Data Imputation</i>	98
	4.9.4	<i>Measurement Error</i>	102
	4.9.5	<i>Outliers and Influential Observations</i>	104
4.10	Summary and Conclusions	107	

## **CHAPTER 5 Hypothesis Tests and Model Selection 113**

5.1	Introduction	113	
5.2	Hypothesis Testing Methodology	113	
	5.2.1	<i>Restrictions and Hypotheses</i>	114
	5.2.2	<i>Nested Models</i>	115
	5.2.3	<i>Testing Procedures</i>	116
	5.2.4	<i>Size, Power, and Consistency of a Test</i>	116
	5.2.5	<i>A Methodological Dilemma: Bayesian Versus Classical Testing</i>	117
5.3	Three Approaches to Testing Hypotheses	117	
	5.3.1	<i>Wald Tests Based on the Distance Measure</i>	120
		5.3.1.a <i>Testing a Hypothesis About a Coefficient</i>	120
		5.3.1.b <i>The F Statistic</i>	123
	5.3.2	<i>Tests Based on the Fit of the Regression</i>	126
		5.3.2.a <i>The Restricted Least Squares Estimator</i>	126
		5.3.2.b <i>The Loss of Fit from Restricted Least Squares</i>	127
		5.3.2.c <i>Testing the Significance of the Regression</i>	129
		5.3.2.d <i>Solving Out the Restrictions and a Caution about <math>R^2</math></i>	129
	5.3.3	<i>Lagrange Multiplier Tests</i>	130
5.4	Large-Sample Tests and Robust Inference	133	
5.5	Testing Nonlinear Restrictions	136	
5.6	Choosing Between Nonnested Models	138	
	5.6.1	<i>Testing Nonnested Hypotheses</i>	139
	5.6.2	<i>An Encompassing Model</i>	140
	5.6.3	<i>Comprehensive Approach—The J Test</i>	140
5.7	A Specification Test	141	
5.8	Model Building—A General to Simple Strategy	143	
	5.8.1	<i>Model Selection Criteria</i>	143
	5.8.2	<i>Model Selection</i>	144

5.8.3	<i>Classical Model Selection</i>	145
5.8.4	<i>Bayesian Model Averaging</i>	145
5.9	Summary and Conclusions	147

## **CHAPTER 6 Functional Form, Difference in Differences, and Structural Change 153**

6.1	Introduction	153
6.2	Using Binary Variables	153
6.2.1	<i>Binary Variables in Regression</i>	153
6.2.2	<i>Several Categories</i>	157
6.2.3	<i>Modeling Individual Heterogeneity</i>	158
6.2.4	<i>Sets of Categories</i>	162
6.2.5	<i>Threshold Effects and Categorical Variables</i>	163
6.2.6	<i>Transition Tables</i>	164
6.3	Difference in Differences Regression	167
6.3.1	<i>Treatment Effects</i>	167
6.3.2	<i>Examining the Effects of Discrete Policy Changes</i>	172
6.4	Using Regression Kinks and Discontinuities to Analyze Social Policy	176
6.4.1	<i>Regression Kinked Design</i>	176
6.4.2	<i>Regression Discontinuity Design</i>	179
6.5	Nonlinearity in the Variables	183
6.5.1	<i>Functional Forms</i>	183
6.5.2	<i>Interaction Effects</i>	185
6.5.3	<i>Identifying Nonlinearity</i>	186
6.5.4	<i>Intrinsically Linear Models</i>	188
6.6	Structural Break and Parameter Variation	191
6.6.1	<i>Different Parameter Vectors</i>	191
6.6.2	<i>Robust Tests of Structural Break with Unequal Variances</i>	193
6.6.3	<i>Pooling Regressions</i>	195
6.7	Summary And Conclusions	197

## **CHAPTER 7 Nonlinear, Semiparametric, and Nonparametric Regression Models 202**

7.1	Introduction	202
7.2	Nonlinear Regression Models	203
7.2.1	<i>Assumptions of the Nonlinear Regression Model</i>	203
7.2.2	<i>The Nonlinear Least Squares Estimator</i>	205
7.2.3	<i>Large-Sample Properties of the Nonlinear Least Squares Estimator</i>	207
7.2.4	<i>Robust Covariance Matrix Estimation</i>	210
7.2.5	<i>Hypothesis Testing and Parametric Restrictions</i>	211

	72.6	<i>Applications</i>	212
	72.7	<i>Loglinear Models</i>	215
	72.8	<i>Computing the Nonlinear Least Squares Estimator</i>	222
7.3		Median and Quantile Regression	225
	7.3.1	<i>Least Absolute Deviations Estimation</i>	226
	7.3.2	<i>Quantile Regression Models</i>	228
7.4		Partially Linear Regression	234
7.5		Nonparametric Regression	235
7.6		Summary and Conclusions	238

## **CHAPTER 8 Endogeneity and Instrumental Variable Estimation 242**

8.1		Introduction	242
8.2		Assumptions of the Extended Model	246
8.3		Instrumental Variables Estimation	248
	8.3.1	<i>Least Squares</i>	248
	8.3.2	<i>The Instrumental Variables Estimator</i>	249
	8.3.3	<i>Estimating the Asymptotic Covariance Matrix</i>	250
	8.3.4	<i>Motivating the Instrumental Variables Estimator</i>	251
8.4		Two-Stage Least Squares, Control Functions, and Limited Information Maximum Likelihood	256
	8.4.1	<i>Two-Stage Least Squares</i>	257
	8.4.2	<i>A Control Function Approach</i>	259
	8.4.3	<i>Limited Information Maximum Likelihood</i>	261
8.5		Endogenous Dummy Variables: Estimating Treatment Effects	262
	8.5.1	<i>Regression Analysis of Treatment Effects</i>	266
	8.5.2	<i>Instrumental Variables</i>	267
	8.5.3	<i>A Control Function Estimator</i>	269
	8.5.4	<i>Propensity Score Matching</i>	270
8.6		Hypothesis Tests	274
	8.6.1	<i>Testing Restrictions</i>	274
	8.6.2	<i>Specification Tests</i>	275
	8.6.3	<i>Testing for Endogeneity: The Hausman and Wu Specification Tests</i>	276
	8.6.4	<i>A Test for Overidentification</i>	277
8.7		Weak Instruments and LIML	279
8.8		Measurement Error	281
	8.8.1	<i>Least Squares Attenuation</i>	282
	8.8.2	<i>Instrumental Variables Estimation</i>	284
	8.8.3	<i>Proxy Variables</i>	285
8.9		Nonlinear Instrumental Variables Estimation	288
8.10		Natural Experiments and the Search for Causal Effects	291
8.11		Summary and Conclusions	295

## Part II Generalized Regression Model and Equation Systems

<b>CHAPTER 9</b>	<b>The Generalized Regression Model and Heteroscedasticity</b>	<b>297</b>
9.1	Introduction	297
9.2	Robust Least Squares Estimation and Inference	298
9.3	Properties of Least Squares and Instrumental Variables	301
9.3.1	<i>Finite-Sample Properties of Least Squares</i>	301
9.3.2	<i>Asymptotic Properties of Least Squares</i>	302
9.3.3	<i>Heteroscedasticity and <math>\text{Var}[\mathbf{b} \mathbf{X}]</math></i>	304
9.3.4	<i>Instrumental Variable Estimation</i>	305
9.4	Efficient Estimation by Generalized Least Squares	306
9.4.1	<i>Generalized Least Squares (GLS)</i>	306
9.4.2	<i>Feasible Generalized Least Squares (FGLS)</i>	309
9.5	Heteroscedasticity and Weighted Least Squares	310
9.5.1	<i>Weighted Least Squares</i>	311
9.5.2	<i>Weighted Least Squares with Known <math>\mathbf{\Omega}</math></i>	311
9.5.3	<i>Estimation When <math>\mathbf{\Omega}</math> Contains Unknown Parameters</i>	312
9.6	Testing for Heteroscedasticity	313
9.6.1	<i>White's General Test</i>	314
9.6.2	<i>The Lagrange Multiplier Test</i>	314
9.7	Two Applications	315
9.7.1	<i>Multiplicative Heteroscedasticity</i>	315
9.7.2	<i>Groupwise Heteroscedasticity</i>	317
9.8	Summary and Conclusions	320
<b>CHAPTER 10</b>	<b>Systems of Regression Equations</b>	<b>326</b>
10.1	Introduction	326
10.2	The Seemingly Unrelated Regressions Model	328
10.2.1	<i>Ordinary Least Squares And Robust Inference</i>	330
10.2.2	<i>Generalized Least Squares</i>	332
10.2.3	<i>Feasible Generalized Least Squares</i>	333
10.2.4	<i>Testing Hypotheses</i>	334
10.2.5	<i>The Pooled Model</i>	336
10.3	Systems of Demand Equations: Singular Systems	339
10.3.1	<i>Cobb–Douglas Cost Function</i>	339
10.3.2	<i>Flexible Functional Forms: The Translog Cost Function</i>	342
10.4	Simultaneous Equations Models	346
10.4.1	<i>Systems of Equations</i>	347
10.4.2	<i>A General Notation for Linear Simultaneous Equations Models</i>	350
10.4.3	<i>The Identification Problem</i>	353
10.4.4	<i>Single Equation Estimation and Inference</i>	358
10.4.5	<i>System Methods of Estimation</i>	362
10.5	Summary and Conclusions	365

<b>CHAPTER 11</b>	<b>Models for Panel Data</b>	<b>373</b>
11.1	Introduction	373
11.2	Panel Data Modeling	374
	11.2.1	<i>General Modeling Framework for Analyzing Panel Data</i> 375
	11.2.2	<i>Model Structures</i> 376
	11.2.3	<i>Extensions</i> 377
	11.2.4	<i>Balanced and Unbalanced Panels</i> 377
	11.2.5	<i>Attrition and Unbalanced Panels</i> 378
	11.2.6	<i>Well-Behaved Panel Data</i> 382
11.3	The Pooled Regression Model	383
	11.3.1	<i>Least Squares Estimation of the Pooled Model</i> 383
	11.3.2	<i>Robust Covariance Matrix Estimation and Bootstrapping</i> 384
	11.3.3	<i>Clustering and Stratification</i> 386
	11.3.4	<i>Robust Estimation Using Group Means</i> 388
	11.3.5	<i>Estimation with First Differences</i> 389
	11.3.6	<i>The Within- and Between-Groups Estimators</i> 390
11.4	The Fixed Effects Model	393
	11.4.1	<i>Least Squares Estimation</i> 393
	11.4.2	<i>A Robust Covariance Matrix for <math>\mathbf{b}_{LSDV}</math></i> 396
	11.4.3	<i>Testing the Significance of the Group Effects</i> 397
	11.4.4	<i>Fixed Time and Group Effects</i> 398
	11.4.5	<i>Reinterpreting the Within Estimator: Instrumental Variables and Control Functions</i> 399
	11.4.6	<i>Parameter Heterogeneity</i> 401
11.5	Random Effects	404
	11.5.1	<i>Least Squares Estimation</i> 405
	11.5.2	<i>Generalized Least Squares</i> 407
	11.5.3	<i>Feasible Generalized Least Squares Estimation of the Random Effects Model when <math>\Sigma</math> is Unknown</i> 408
	11.5.4	<i>Robust Inference and Feasible Generalized Least Squares</i> 409
	11.5.5	<i>Testing for Random Effects</i> 410
	11.5.6	<i>Hausman's Specification Test for the Random Effects Model</i> 414
	11.5.7	<i>Extending the Unobserved Effects Model: Mundlak's Approach</i> 415
	11.5.8	<i>Extending the Random and Fixed Effects Models: Chamberlain's Approach</i> 416
11.6	Nonspherical Disturbances and Robust Covariance Matrix Estimation	421
	11.6.1	<i>Heteroscedasticity in the Random Effects Model</i> 421
	11.6.2	<i>Autocorrelation in Panel Data Models</i> 422
11.7	Spatial Autocorrelation	422

11.8	Endogeneity	427	
	11.8.1	<i>Instrumental Variable Estimation</i>	427
	11.8.2	<i>Hausman and Taylor's Instrumental Variables Estimator</i>	429
	11.8.3	<i>Consistent Estimation of Dynamic Panel Data Models: Anderson and Hsiao's Iv Estimator</i>	433
	11.8.4	<i>Efficient Estimation of Dynamic Panel Data Models: The Arellano/Bond Estimators</i>	436
	11.8.5	<i>Nonstationary Data and Panel Data Models</i>	445
11.9	Nonlinear Regression with Panel Data	446	
	11.9.1	<i>A Robust Covariance Matrix for Nonlinear Least Squares</i>	446
	11.9.2	<i>Fixed Effects in Nonlinear Regression Models</i>	447
	11.9.3	<i>Random Effects</i>	449
11.10	Parameter Heterogeneity	450	
	11.10.1	<i>A Random Coefficients Model</i>	450
	11.10.2	<i>A Hierarchical Linear Model</i>	453
	11.10.3	<i>Parameter Heterogeneity and Dynamic Panel Data Models</i>	455
11.11	Summary and Conclusions	459	

### **Part III Estimation Methodology**

#### **CHAPTER 12 Estimation Frameworks in Econometrics 465**

12.1	Introduction	465	
12.2	Parametric Estimation and Inference	467	
	12.2.1	<i>Classical Likelihood-Based Estimation</i>	467
	12.2.2	<i>Modeling Joint Distributions with Copula Functions</i>	469
12.3	Semiparametric Estimation	472	
	12.3.1	<i>Gmm Estimation in Econometrics</i>	473
	12.3.2	<i>Maximum Empirical Likelihood Estimation</i>	473
	12.3.3	<i>Least Absolute Deviations Estimation and Quantile Regression</i>	475
	12.3.4	<i>Kernel Density Methods</i>	475
	12.3.5	<i>Comparing Parametric and Semiparametric Analyses</i>	476
12.4	Nonparametric Estimation	478	
	12.4.1	<i>Kernel Density Estimation</i>	478
12.5	Properties of Estimators	481	
	12.5.1	<i>Statistical Properties of Estimators</i>	481
	12.5.2	<i>Extremum Estimators</i>	482
	12.5.3	<i>Assumptions for Asymptotic Properties of Extremum Estimators</i>	483
	12.5.4	<i>Asymptotic Properties of Estimators</i>	485
	12.5.5	<i>Testing Hypotheses</i>	487
12.6	Summary and Conclusions	487	

## **CHAPTER 13 Minimum Distance Estimation and the Generalized Method of Moments 488**

13.1	Introduction	488	
13.2	Consistent Estimation: The Method of Moments	489	
	13.2.1	<i>Random Sampling and Estimating the Parameters of Distributions</i>	490
	13.2.2	<i>Asymptotic Properties of the Method of Moments Estimator</i>	493
	13.2.3	<i>Summary—The Method of Moments</i>	496
13.3	Minimum Distance Estimation	496	
13.4	The Generalized Method of Moments (GMM) Estimator	500	
	13.4.1	<i>Estimation Based on Orthogonality Conditions</i>	501
	13.4.2	<i>Generalizing the Method of Moments</i>	502
	13.4.3	<i>Properties of the GMM Estimator</i>	506
13.5	Testing Hypotheses in the GMM Framework	510	
	13.5.1	<i>Testing the Validity of the Moment Restrictions</i>	510
	13.5.2	<i>Gmm Wald Counterparts to the WALD, LM, and LR Tests</i>	512
13.6	Gmm Estimation of Econometric Models	513	
	13.6.1	<i>Single-Equation Linear Models</i>	514
	13.6.2	<i>Single-Equation Nonlinear Models</i>	519
	13.6.3	<i>Seemingly Unrelated Regression Equations</i>	522
	13.6.4	<i>Gmm Estimation of Dynamic Panel Data Models</i>	523
13.7	Summary and Conclusions	534	

## **CHAPTER 14 Maximum Likelihood Estimation 537**

14.1	Introduction	537	
14.2	The Likelihood Function and Identification of the Parameters	537	
14.3	Efficient Estimation: The Principle of Maximum Likelihood	539	
14.4	Properties of Maximum Likelihood Estimators	541	
	14.4.1	<i>Regularity Conditions</i>	542
	14.4.2	<i>Properties of Regular Densities</i>	543
	14.4.3	<i>The Likelihood Equation</i>	544
	14.4.4	<i>The Information Matrix Equality</i>	545
	14.4.5	<i>Asymptotic Properties of the Maximum Likelihood Estimator</i>	545
	14.4.5.a	<i>Consistency</i>	545
	14.4.5.b	<i>Asymptotic Normality</i>	547
	14.4.5.c	<i>Asymptotic Efficiency</i>	548
	14.4.5.d	<i>Invariance</i>	548
	14.4.5.e	<i>Conclusion</i>	549
	14.4.6	<i>Estimating the Asymptotic Variance of the Maximum Likelihood Estimator</i>	549
14.5	Conditional Likelihoods and Econometric Models	551	

14.6	Hypothesis and Specification Tests and Fit Measures	552
14.6.1	<i>The Likelihood Ratio Test</i>	554
14.6.2	<i>The Wald Test</i>	555
14.6.3	<i>The Lagrange Multiplier Test</i>	557
14.6.4	<i>An Application of the Likelihood-Based Test Procedures</i>	558
14.6.5	<i>Comparing Models and Computing Model Fit</i>	560
14.6.6	<i>Vuong's Test and the Kullback–Leibler Information Criterion</i>	562
14.7	Two-Step Maximum Likelihood Estimation	564
14.8	Pseudo-Maximum Likelihood Estimation and Robust Asymptotic Covariance Matrices	570
14.8.1	<i>A Robust Covariance Matrix Estimator for the MLE</i>	570
14.8.2	<i>Cluster Estimators</i>	573
14.9	Maximum Likelihood Estimation of Linear Regression Models	576
14.9.1	<i>Linear Regression Model with Normally Distributed Disturbances</i>	576
14.9.2	<i>Some Linear Models with Nonnormal Disturbances</i>	578
14.9.3	<i>Hypothesis Tests for Regression Models</i>	580
14.10	The Generalized Regression Model	585
14.10.1	<i>GLS With Known <math>\Omega</math></i>	585
14.10.2	<i>Iterated Feasible GLS With Estimated <math>\Omega</math></i>	586
14.10.3	<i>Multiplicative Heteroscedasticity</i>	586
14.10.4	<i>The Method of Scoring</i>	587
14.11	Nonlinear Regression Models and Quasi-Maximum Likelihood Estimation	591
14.11.1	<i>Maximum Likelihood Estimation</i>	592
14.11.2	<i>Quasi-Maximum Likelihood Estimation</i>	595
14.12	Systems of Regression Equations	600
14.12.1	<i>The Pooled Model</i>	600
14.12.2	<i>The SUR Model</i>	601
14.13	Simultaneous Equations Models	604
14.14	Panel Data Applications	605
14.14.1	<i>ML Estimation of the Linear Random Effects Model</i>	606
14.14.2	<i>Nested Random Effects</i>	609
14.14.3	<i>Clustering Over More than One Level</i>	612
14.14.4	<i>Random Effects in Nonlinear Models: MLE Using Quadrature</i>	613
14.14.5	<i>Fixed Effects in Nonlinear Models: The Incidental Parameters Problem</i>	617
14.15	Latent Class and Finite Mixture Models	622
14.15.1	<i>A Finite Mixture Model</i>	622
14.15.2	<i>Modeling the Class Probabilities</i>	624



14.15.3	<i>Latent Class Regression Models</i>	625
14.15.4	<i>Predicting Class Membership and <math>\mathbf{B}_i</math></i>	626
14.15.5	<i>Determining the Number of Classes</i>	628
14.15.6	<i>A Panel Data Application</i>	628
14.15.7	<i>A Semiparametric Random Effects Model</i>	633
14.16	Summary and Conclusions	635

## **CHAPTER 15 Simulation-Based Estimation and Inference and Random Parameter Models 641**

15.1	Introduction	641
15.2	Random Number Generation	643
15.2.1	<i>Generating Pseudo-Random Numbers</i>	643
15.2.2	<i>Sampling from a Standard Uniform Population</i>	644
15.2.3	<i>Sampling from Continuous Distributions</i>	645
15.2.4	<i>Sampling from a Multivariate Normal Population</i>	646
15.2.5	<i>Sampling from Discrete Populations</i>	646
15.3	Simulation-Based Statistical Inference: The Method of Krinsky and Robb	647
15.4	Bootstrapping Standard Errors and Confidence Intervals	650
15.4.1	<i>Types of Bootstraps</i>	651
15.4.2	<i>Bias Reduction with Bootstrap Estimators</i>	651
15.4.3	<i>Bootstrapping Confidence Intervals</i>	652
15.4.4	<i>Bootstrapping with Panel Data: The Block Bootstrap</i>	652
15.5	Monte Carlo Studies	653
15.5.1	<i>A Monte Carlo Study: Behavior of a Test Statistic</i>	655
15.5.2	<i>A Monte Carlo Study: The Incidental Parameters Problem</i>	656
15.6	Simulation-Based Estimation	660
15.6.1	<i>Random Effects in a Nonlinear Model</i>	661
15.6.2	<i>Monte Carlo Integration</i>	662
15.6.2a	<i>Halton Sequences and Random Draws for Simulation-Based Integration</i>	664
15.6.2b	<i>Computing Multivariate Normal Probabilities Using the GHK Simulator</i>	666
15.6.3	<i>Simulation-Based Estimation of Random Effects Models</i>	668
15.7	A Random Parameters Linear Regression Model	673
15.8	Hierarchical Linear Models	678
15.9	Nonlinear Random Parameter Models	680
15.10	Individual Parameter Estimates	681
15.11	Mixed Models and Latent Class Models	689
15.12	Summary and Conclusions	692

<b>CHAPTER 16 Bayesian Estimation and Inference</b>	<b>694</b>
16.1 Introduction	694
16.2 Bayes' Theorem and the Posterior Density	695
16.3 Bayesian Analysis of the Classical Regression Model	697
16.3.1 Analysis with a Noninformative Prior	698
16.3.2 Estimation with an Informative Prior Density	700
16.4 Bayesian Inference	703
16.4.1 Point Estimation	703
16.4.2 Interval Estimation	704
16.4.3 Hypothesis Testing	705
16.4.4 Large-Sample Results	707
16.5 Posterior Distributions and the Gibbs Sampler	707
16.6 Application: Binomial Probit Model	710
16.7 Panel Data Application: Individual Effects Models	713
16.8 Hierarchical Bayes Estimation of a Random Parameters Model	715
16.9 Summary and Conclusions	721

## **Part IV Cross Sections, Panel Data, and Microeconometrics**

<b>CHAPTER 17 Binary Outcomes and Discrete Choices</b>	<b>725</b>
17.1 Introduction	725
17.2 Models for Binary Outcomes	728
17.2.1 Random Utility	729
17.2.2 The Latent Regression Model	730
17.2.3 Functional Form and Probability	731
17.2.4 Partial Effects in Binary Choice Models	734
17.2.5 Odds Ratios in Logit Models	736
17.2.6 The Linear Probability Model	740
17.3 Estimation and Inference for Binary Choice Models	742
17.3.1 Robust Covariance Matrix Estimation	744
17.3.2 Hypothesis Tests	746
17.3.3 Inference for Partial Effects	749
17.3.3.a The Delta Method	749
17.3.3.b An Adjustment to the Delta Method	751
17.3.3.c The Method of Krinsky and Robb	752
17.3.3.d Bootstrapping	752
17.3.4 Interaction Effects	755
17.4 Measuring Goodness of Fit for Binary Choice Models	757
17.4.1 Fit Measures Based on the Fitting Criterion	757
17.4.2 Fit Measures Based on Predicted Values	758
17.4.3 Summary of Fit Measures	760
17.5 Specification Analysis	762
17.5.1 Omitted Variables	763

175.2	<i>Heteroscedasticity</i>	764
175.3	<i>Distributional Assumptions</i>	766
175.4	<i>Choice-Based Sampling</i>	768
17.6	Treatment Effects and Endogenous Variables in Binary Choice Models	769
176.1	<i>Endogenous Treatment Effect</i>	770
176.2	<i>Endogenous Continuous Variable</i>	773
176.2.a	<i>IV and GMM Estimation</i>	773
176.2.b	<i>Partial ML Estimation</i>	774
176.2.c	<i>Full Information Maximum Likelihood Estimation</i>	774
176.2.d	<i>Residual Inclusion and Control Functions</i>	775
176.2.e	<i>A Control Function Estimator</i>	775
176.3	<i>Endogenous Sampling</i>	777
17.7	Panel Data Models	780
177.1	<i>The Pooled Estimator</i>	781
177.2	<i>Random Effects</i>	782
177.3	<i>Fixed Effects</i>	785
177.3.a	<i>A Conditional Fixed Effects Estimator</i>	787
177.3.b	<i>Mundlak's Approach, Variable Addition, and Bias Reduction</i>	792
177.4	<i>Dynamic Binary Choice Models</i>	794
177.5	<i>A Semiparametric Model for Individual Heterogeneity</i>	797
177.6	<i>Modeling Parameter Heterogeneity</i>	798
177.7	<i>Nonresponse, Attrition, and Inverse Probability Weighting</i>	801
17.9	Spatial Binary Choice Models	804
17.9	The Bivariate Probit Model	807
179.1	<i>Maximum Likelihood Estimation</i>	808
179.2	<i>Testing for Zero Correlation</i>	811
179.3	<i>Partial Effects</i>	811
179.4	<i>A Panel Data Model for Bivariate Binary Response</i>	814
179.5	<i>A Recursive Bivariate Probit Model</i>	815
17.10	A Multivariate Probit Model	819
17.11	Summary and Conclusions	822

## **CHAPTER 18 Multinomial Choices and Event Counts 826**

18.1	Introduction	826
18.2	Models for Unordered Multiple Choices	827
18.2.1	<i>Random Utility Basis of the Multinomial Logit Model</i>	827
18.2.2	<i>The Multinomial Logit Model</i>	829
18.2.3	<i>The Conditional Logit Model</i>	833
18.2.4	<i>The Independence from Irrelevant Alternatives Assumption</i>	834
18.2.5	<i>Alternative Choice Models</i>	835
18.2.5.a	<i>Heteroscedastic Extreme Value Model</i>	836

	18.2.5.b	<i>Multinomial Probit Model</i>	836
	18.2.5.c	<i>The Nested Logit Model</i>	837
18.2.6		<i>Modeling Heterogeneity</i>	845
	18.2.6.a	<i>The Mixed Logit Model</i>	845
	18.2.6.b	<i>A Generalized Mixed Logit Model</i>	846
	18.2.6.c	<i>Latent Classes</i>	849
	18.2.6.d	<i>Attribute Nonattendance</i>	851
18.2.7		<i>Estimating Willingness to Pay</i>	853
18.2.8		<i>Panel Data and Stated Choice Experiments</i>	856
	18.2.8.a	<i>The Mixed Logit Model</i>	857
	18.2.8.b	<i>Random Effects and the Nested Logit Model</i>	858
	18.2.8.c	<i>A Fixed Effects Multinomial Logit Model</i>	859
18.2.9		<i>Aggregate Market Share Data—The BLP Random Parameters Model</i>	863
18.3		<i>Random Utility Models for Ordered Choices</i>	865
	18.3.1	<i>The Ordered Probit Model</i>	869
18.3.2.A		<i>Specification Test for the Ordered Choice Model</i>	872
18.3.3		<i>Bivariate Ordered Probit Models</i>	873
18.3.4		<i>Panel Data Applications</i>	875
	18.3.4.a	<i>Ordered Probit Models with Fixed Effects</i>	875
	18.3.4.b	<i>Ordered Probit Models with Random Effects</i>	877
18.3.5		<i>Extensions of the Ordered Probit Model</i>	881
	18.3.5.a	<i>Threshold Models—Generalized Ordered Choice Models</i>	881
	18.3.5.b	<i>Thresholds and Heterogeneity—Anchoring Vignettes</i>	883
18.4		<i>Models for Counts of Events</i>	884
	18.4.1	<i>The Poisson Regression Model</i>	885
	18.4.2	<i>Measuring Goodness of Fit</i>	887
	18.4.3	<i>Testing for Overdispersion</i>	888
	18.4.4	<i>Heterogeneity and the Negative Binomial Regression Model</i>	889
	18.4.5	<i>Functional Forms for Count Data Models</i>	890
	18.4.6	<i>Truncation and Censoring in Models for Counts</i>	894
	18.4.7	<i>Panel Data Models</i>	898
	18.4.7.a	<i>Robust Covariance Matrices for Pooled Estimators</i>	898
	18.4.7.b	<i>Fixed Effects</i>	900
	18.4.7.c	<i>Random Effects</i>	902
	18.4.8	<i>Two-Part Models: Zero-Inflation and Hurdle Models</i>	905
	18.4.9	<i>Endogenous Variables and Endogenous Participation</i>	910
18.5		<i>Summary and Conclusions</i>	914

**CHAPTER 19 Limited Dependent Variables—Truncation, Censoring, and Sample Selection 918**

19.1	<i>Introduction</i>	918
------	---------------------	-----

19.2	Truncation	918
	19.2.1	<i>Truncated Distributions</i> 919
	19.2.2	<i>Moments of Truncated Distributions</i> 920
	19.2.3	<i>The Truncated Regression Model</i> 922
	19.2.4	<i>The Stochastic Frontier Model</i> 924
19.3	Censored Data	930
	19.3.1	<i>The Censored Normal Distribution</i> 931
	19.3.2	<i>The Censored Regression (Tobit) Model</i> 933
	19.3.3	<i>Estimation</i> 936
	19.3.4	<i>Two-Part Models and Corner Solutions</i> 938
	19.3.5	<i>Specification Issues</i> 944
		19.3.5.a <i>Endogenous Right-Hand-Side Variables</i> 944
		19.3.5.b <i>Heteroscedasticity</i> 945
		19.3.5.c <i>Nonnormality</i> 947
	19.3.6	<i>Panel Data Applications</i> 948
19.4	Sample Selection and Incidental Truncation	949
	19.4.1	<i>Incidental Truncation in a Bivariate Distribution</i> 949
	19.4.2	<i>Regression in a Model of Selection</i> 950
	19.4.3	<i>Two-Step and Maximum Likelihood Estimation</i> 953
	19.4.4	<i>Sample Selection in Nonlinear Models</i> 957
	19.4.5	<i>Panel Data Applications of Sample Selection Models</i> 961
		19.4.5.a <i>Common Effects in Sample Selection Models</i> 961
		19.4.5.b <i>Attrition</i> 964
19.5	Models for Duration	965
	19.5.1	<i>Models for Duration Data</i> 966
	19.5.2	<i>Duration Data</i> 966
	19.5.3	<i>A Regression-Like Approach: Parametric Models of Duration</i> 967
		19.5.3.a <i>Theoretical Background</i> 967
		19.5.3.b <i>Models of the Hazard Function</i> 968
		19.5.3.c <i>Maximum Likelihood Estimation</i> 970
		19.5.3.d <i>Exogenous Variables</i> 971
		19.5.3.e <i>Heterogeneity</i> 972
	19.5.4	<i>Nonparametric and Semiparametric Approaches</i> 973
19.6	Summary and Conclusions	976

## **Part V Time Series and Macroeconometrics**

### **CHAPTER 20 Serial Correlation 981**

20.1	Introduction	981
20.2	The Analysis of Time-Series Data	984
20.3	Disturbance Processes	987
	20.3.1	<i>Characteristics of Disturbance Processes</i> 987
	20.3.2	<i>Ar(1) Disturbances</i> 989
20.4	Some Asymptotic Results for Analyzing Time-Series Data	990

20.4.1	<i>Convergence of Moments—The Ergodic Theorem</i>	991
20.4.2	<i>Convergence to Normality—A Central Limit Theorem</i>	994
20.5	Least Squares Estimation	996
20.5.1	<i>Asymptotic Properties of Least Squares</i>	996
20.5.2	<i>Estimating the Variance of the Least Squares Estimator</i>	998
20.6	Gmm Estimation	999
20.7	Testing for Autocorrelation	1000
20.7.1	<i>Lagrange Multiplier Test</i>	1000
20.7.2	<i>Box And Pierce’s Test and Ljung’s Refinement</i>	1001
20.7.3	<i>The Durbin–Watson Test</i>	1001
20.7.4	<i>Testing in the Presence of a Lagged Dependent Variable</i>	1002
20.7.5	<i>Summary of Testing Procedures</i>	1002
20.8	Efficient Estimation when $\Omega$ is Known	1003
20.9	Estimation when $\Omega$ is Unknown	1004
20.9.1	<i>Ar(1) Disturbances</i>	1004
20.9.2	<i>Application: Estimation of a Model with Autocorrelation</i>	1005
20.9.3	<i>Estimation with a Lagged Dependent Variable</i>	1007
20.10	Autoregressive Conditional Heteroscedasticity	1010
20.10.1	<i>The ARCH(1) Model</i>	1011
20.10.2	<i>ARCH(q), ARCH-In-Mean, and Generalized ARCH Models</i>	1012
20.10.3	<i>Maximum Likelihood Estimation of the GARCH Model</i>	1014
20.10.4	<i>Testing for GARCH Effects</i>	1017
20.10.5	<i>Pseudo–Maximum Likelihood Estimation</i>	1018
20.11	Summary and Conclusions	1019
<b>CHAPTER 21 Nonstationary Data 1022</b>		
21.1	Introduction	1022
21.2	Nonstationary Processes and Unit Roots	1022
21.2.1	<i>The Lag and Difference Operators</i>	1022
21.2.2	<i>Integrated Processes and Differencing</i>	1023
21.2.3	<i>Random Walks, Trends, and Spurious Regressions</i>	1026
21.2.4	<i>Tests for Unit Roots in Economic Data</i>	1028
21.2.5	<i>The Dickey–Fuller Tests</i>	1029
21.2.6	<i>The KPSS Test of Stationarity</i>	1038
21.3	Cointegration	1039
21.3.1	<i>Common Trends</i>	1043
21.3.2	<i>Error Correction and Var Representations</i>	1044
21.3.3	<i>Testing for Cointegration</i>	1045
21.3.4	<i>Estimating Cointegration Relationships</i>	1048
21.3.5	<i>Application: German Money Demand</i>	1048
	21.3.5.a <i>Cointegration Analysis and a Long-Run Theoretical Model</i>	1049

	21.3.5.b	<i>Testing for Model Instability</i>	1050
21.4		Nonstationary Panel Data	1051
21.5		Summary and Conclusions	1052

**References 1054****Index 1098****Part VI Online Appendices****Appendix A Matrix Algebra A-1**

A.1		Terminology	A-1
A.2		Algebraic Manipulation of Matrices	A-2
	A.2.1	<i>Equality of Matrices</i>	A-2
	A.2.2	<i>Transposition</i>	A-2
	A.2.3	<i>Vectorization</i>	A-3
	A.2.4	<i>Matrix Addition</i>	A-3
	A.2.5	<i>Vector Multiplication</i>	A-3
	A.2.6	<i>A Notation for Rows and Columns of a Matrix</i>	A-3
	A.2.7	<i>Matrix Multiplication and Scalar Multiplication</i>	A-4
	A.2.8	<i>Sums of Values</i>	A-5
	A.2.9	<i>A Useful Idempotent Matrix</i>	A-6
A.3		Geometry of Matrices	A-8
	A.3.1	<i>Vector Spaces</i>	A-8
	A.3.2	<i>Linear Combinations of Vectors and Basis Vectors</i>	A-9
	A.3.3	<i>Linear Dependence</i>	A-11
	A.3.4	<i>Subspaces</i>	A-12
	A.3.5	<i>Rank of a Matrix</i>	A-12
	A.3.6	<i>Determinant of a Matrix</i>	A-15
	A.3.7	<i>A Least Squares Problem</i>	A-16
A.4		Solution of a System of Linear Equations	A-19
	A.4.1	<i>Systems of Linear Equations</i>	A-19
	A.4.2	<i>Inverse Matrices</i>	A-19
	A.4.3	<i>Nonhomogeneous Systems of Equations</i>	A-21
	A.4.4	<i>Solving the Least Squares Problem</i>	A-21
A.5		Partitioned Matrices	A-22
	A.5.1	<i>Addition and Multiplication of Partitioned Matrices</i>	A-22
	A.5.2	<i>Determinants of Partitioned Matrices</i>	A-23
	A.5.3	<i>Inverses of Partitioned Matrices</i>	A-23
	A.5.4	<i>Deviations From Means</i>	A-23
	A.5.5	<i>Kronecker Products</i>	A-24
A.6		Characteristic Roots And Vectors	A-24
	A.6.1	<i>The Characteristic Equation</i>	A-25
	A.6.2	<i>Characteristic Vectors</i>	A-25
	A.6.3	<i>General Results for Characteristic Roots And Vectors</i>	A-26

A.6.4	<i>Diagonalization and Spectral Decomposition of a Matrix</i>	A-26
A.6.5	<i>Rank of a Matrix</i>	A-27
A.6.6	<i>Condition Number of a Matrix</i>	A-28
A.6.7	<i>Trace of a Matrix</i>	A-29
A.6.8	<i>Determinant of a Matrix</i>	A-30
A.6.9	<i>Powers of a Matrix</i>	A-30
A.6.10	<i>Idempotent Matrices</i>	A-32
A.6.11	<i>Factoring a Matrix: The Cholesky Decomposition</i>	A-32
A.6.12	<i>Singular Value Decomposition</i>	A-33
A.6.13	<i>Qr Decomposition</i>	A-33
A.6.14	<i>The Generalized Inverse of a Matrix</i>	A-33
A.7	<i>Quadratic Forms And Definite Matrices</i>	A-34
A.7.1	<i>Nonnegative Definite Matrices</i>	A-35
A.7.2	<i>Idempotent Quadratic Forms</i>	A-36
A.7.3	<i>Comparing Matrices</i>	A-37
A.8	<i>Calculus And Matrix Algebra</i>	A-37
A.8.1	<i>Differentiation and the Taylor Series</i>	A-37
A.8.2	<i>Optimization</i>	A-41
A.8.3	<i>Constrained Optimization</i>	A-43
A.8.4	<i>Transformations</i>	A-45

## Appendix B Probability and Distribution

### Theory B-1

B.1	<i>Introduction</i>	B-1
B.2	<i>Random Variables</i>	B-1
B.2.1	<i>Probability Distributions</i>	B-2
B.2.2	<i>Cumulative Distribution Function</i>	B-2
B.3	<i>Expectations of a Random Variable</i>	B-3
B.4	<i>Some Specific Probability Distributions</i>	B-6
B.4.1	<i>The Normal and Skew Normal Distributions</i>	B-6
B.4.2	<i>The Chi-Squared, t, and F Distributions</i>	B-8
B.4.3	<i>Distributions with Large Degrees of Freedom</i>	B-11
B.4.4	<i>Size Distributions: The Lognormal Distribution</i>	B-12
B.4.5	<i>The Gamma and Exponential Distributions</i>	B-13
B.4.6	<i>The Beta Distribution</i>	B-13
B.4.7	<i>The Logistic Distribution</i>	B-14
B.4.8	<i>The Wishart Distribution</i>	B-14
B.4.9	<i>Discrete Random Variables</i>	B-15
B.5	<i>The Distribution of a Function of a Random Variable</i>	B-15
B.6	<i>Representations of a Probability Distribution</i>	B-18
B.7	<i>Joint Distributions</i>	B-19
B.7.1	<i>Marginal Distributions</i>	B-20
B.7.2	<i>Expectations in a Joint Distribution</i>	B-20
B.7.3	<i>Covariance and Correlation</i>	B-21



	B.7.4	<i>Distribution of a Function of Bivariate Random Variables</i>	B-22
B.8		Conditioning in a Bivariate Distribution	B-23
	B.8.1	<i>Regression: The Conditional Mean</i>	B-24
	B.8.2	<i>Conditional Variance</i>	B-24
	B.8.3	<i>Relationships among Marginal and Conditional Moments</i>	B-24
	B.8.4	<i>The Analysis of Variance</i>	B-26
	B.8.5	<i>Linear Projection</i>	B-27
B.9		The Bivariate Normal Distribution	B-28
B.10		Multivariate Distributions	B-29
	B.10.1	<i>Moments</i>	B-29
	B.10.2	<i>Sets of Linear Functions</i>	B-30
	B.10.3	<i>Nonlinear Functions: The Delta Method</i>	B-31
B.11		The Multivariate Normal Distribution	B-31
	B.11.1	<i>Marginal and Conditional Normal Distributions</i>	B-32
	B.11.2	<i>The Classical Normal Linear Regression Model</i>	B-33
	B.11.3	<i>Linear Functions of a Normal Vector</i>	B-33
	B.11.4	<i>Quadratic Forms in a Standard Normal Vector</i>	B-34
	B.11.5	<i>The F Distribution</i>	B-36
	B.11.6	<i>A Full Rank Quadratic Form</i>	B-36
	B.11.7	<i>Independence of a Linear and a Quadratic Form</i>	B-38
<b>Appendix C Estimation and Inference C-1</b>			
	C.1	Introduction	C-1
	C.2	Samples and Random Sampling	C-1
	C.3	Descriptive Statistics	C-2
	C.4	Statistics as Estimators—Sampling Distributions	C-6
	C.5	Point Estimation of Parameters	C-9
		C.5.1 <i>Estimation in a Finite Sample</i>	C-9
		C.5.2 <i>Efficient Unbiased Estimation</i>	C-12
	C.6	Interval Estimation	C-14
	C.7	Hypothesis Testing	C-16
		C.7.1 <i>Classical Testing Procedures</i>	C-16
		C.7.2 <i>Tests Based on Confidence Intervals</i>	C-19
		C.7.3 <i>Specification Tests</i>	D-1
<b>Appendix D Large-Sample Distribution Theory D-1</b>			
	D.1	Introduction	D-1
	D.2	Large-Sample Distribution Theory 1	D-2
		D.2.1 <i>Convergence in Probability</i>	D-2
		D.2.2 <i>Other forms of Convergence and Laws of Large Numbers</i>	D-5
		D.2.3 <i>Convergence of Functions</i>	D-9
		D.2.4 <i>Convergence to a Random Variable</i>	D-10

	D.2.5	<i>Convergence in Distribution: Limiting Distributions</i>	D-11
	D.2.6	<i>Central Limit Theorems</i>	D-14
	D.2.7	<i>The Delta Method</i>	D-19
D.3		Asymptotic Distributions	D-19
	D.3.1	<i>Asymptotic Distribution of a Nonlinear Function</i>	D-21
	D.3.2	<i>Asymptotic Expectations</i>	D-22
D.4		Sequences and the Order of a Sequence	D-24
<b>Appendix E Computation and Optimization E-1</b>			
E.1		Introduction	E-1
E.2		Computation in Econometrics	E-1
	E.2.1	<i>Computing Integrals</i>	E-2
	E.2.2	<i>The Standard Normal Cumulative Distribution Function</i>	E-2
	E.2.3	<i>The Gamma and Related Functions</i>	E-3
	E.2.4	<i>Approximating Integrals by Quadrature</i>	E-4
E.3		Optimization	E-5
	E.3.1	<i>Algorithms</i>	E-7
	E.3.2	<i>Computing Derivatives</i>	E-7
	E.3.3	<i>Gradient Methods</i>	E-9
	E.3.4	<i>Aspects of Maximum Likelihood Estimation</i>	E-12
	E.3.5	<i>Optimization with Constraints</i>	E-14
	E.3.6	<i>Some Practical Considerations</i>	E-15
	E.3.7	<i>The EM Algorithm</i>	E-17
E.4		Examples	E-19
	E.4.1	<i>Function of one Parameter</i>	E-19
	E.4.2	<i>Function of two Parameters: The Gamma Distribution</i>	E-20
	E.4.3	<i>A Concentrated Log-Likelihood Function</i>	E-21
<b>Appendix F Data Sets Used in Applications F-1</b>			

## EXAMPLES AND APPLICATIONS



### **CHAPTER 1 Econometrics 1**

Example 1.1	Behavioral Models and the Nobel Laureates	2
Example 1.2	Keynes's Consumption Function	5

### **CHAPTER 2 The Linear Regression Model 12**

Example 2.1	Keynes's Consumption Function	14
Example 2.2	Earnings and Education	15
Example 2.3	The U.S. Gasoline Market	19
Example 2.4	The Translog Model	19
Example 2.5	Short Rank	20
Example 2.6	An Inestimable Model	21
Example 2.7	Nonzero Conditional Mean of the Disturbances	22

### **CHAPTER 3 Least Squares Regression 28**

Example 3.1	Partial Correlations	41
Example 3.2	Fit of a Consumption Function	44
Example 3.3	Analysis of Variance for the Investment Equation	44
Example 3.4	Art Appreciation	48

### **CHAPTER 4 Estimating the Regression Model by Least Squares 54**

Example 4.1	The Sampling Distribution of a Least Squares Estimator	58
Example 4.2	Omitted Variable in a Demand Equation	59
Example 4.3	Least Squares Vs. Least Absolute Deviations—A Monte Carlo Study	68
Example 4.4	Linear Projection: A Sampling Experiment	72
Example 4.5	Robust Inference about the Art Market	76
Example 4.6	Clustering and Block Bootstrapping	78
Example 4.7	Nonlinear Functions of Parameters: The Delta Method	80
Example 4.8	Confidence Interval for the Income Elasticity of Demand for Gasoline	83
Example 4.9	Oaxaca Decomposition of Home Sale Prices	85
Example 4.10	Pricing Art	90
Example 4.11	Multicollinearity in the Longley Data	95
Example 4.12	Predicting Movie Success	97
Example 4.13	Imputation in the Survey of Consumer Finances	101

**CHAPTER 5 Hypothesis Tests and Model Selection 113**

Example 5.1	Art Appreciation	121
Example 5.2	Earnings Equation	122
Example 5.3	Restricted Investment Equation	124
Example 5.4	F Test for the Earnings Equation	129
Example 5.5	Production Functions	130
Example 5.6	A Long-Run Marginal Propensity to Consume	137
Example 5.7	J Test for a Consumption Function	141
Example 5.8	Size of a RESET Test	142
Example 5.9	Bayesian Averaging of Classical Estimates	147

**CHAPTER 6 Functional Form, Difference in Differences, and Structural Change 153**

Example 6.1	Dummy Variable in an Earnings Equation	154
Example 6.2	Value of a Signature	155
Example 6.3	Gender and Time Effects in a Log Wage Equation	156
Example 6.4	Genre Effects on Movie Box Office Receipts	158
Example 6.5	Sports Economics: Using Dummy Variables for Unobserved Heterogeneity 5	160
Example 6.6	Analysis of Covariance	162
Example 6.7	Education Thresholds in a Log Wage Equation	165
Example 6.8	SAT Scores	169
Example 6.9	A Natural Experiment: The Mariel Boatlift	169
Example 6.10	Effect of the Minimum Wage	170
Example 6.11	Difference in Differences Analysis of a Price Fixing Conspiracy 13	172
Example 6.12	Policy Analysis Using Kinked Regressions	178
Example 6.13	The Treatment Effect of Compulsory Schooling	180
Example 6.14	Interest Elasticity of Mortgage Demand	180
Example 6.15	Quadratic Regression	184
Example 6.16	Partial Effects in a Model with Interactions	186
Example 6.17	Functional Form for a Nonlinear Cost Function	187
Example 6.18	Intrinsically Linear Regression	189
Example 6.19	CES Production Function	190
Example 6.20	Structural Break in the Gasoline Market	192
Example 6.21	Sample Partitioning by Gender	194
Example 6.22	The World Health Report	194
Example 6.23	Pooling in a Log Wage Model	196

**CHAPTER 7 Nonlinear, Semiparametric, and Nonparametric Regression Models 202**

Example 7.1	CES Production Function	203
Example 7.2	Identification in a Translog Demand System	204
Example 7.3	First-Order Conditions for a Nonlinear Model	206
Example 7.4	Analysis of a Nonlinear Consumption Function	213

Example 7.5	The Box–Cox Transformation	214
Example 7.6	Interaction Effects in a Loglinear Model for Income	216
Example 7.7	Generalized Linear Models for the Distribution of Healthcare Costs	221
Example 7.8	Linearized Regression	223
Example 7.9	Nonlinear Least Squares	224
Example 7.10	LAD Estimation of a Cobb–Douglas Production Function	228
Example 7.11	Quantile Regression for Smoking Behavior	230
Example 7.12	Income Elasticity of Credit Card Expenditures	231
Example 7.13	Partially Linear Translog Cost Function	235
Example 7.14	A Nonparametric Average Cost Function	237

## **CHAPTER 8 Endogeneity and Instrumental Variable Estimation 242**

Example 8.1	Models with Endogenous Right-Hand-Side Variables	242
Example 8.2	Instrumental Variable Analysis	252
Example 8.3	Streams as Instruments	254
Example 8.4	Instrumental Variable in Regression	255
Example 8.5	Instrumental Variable Estimation of a Labor Supply Equation	258
Example 8.6	German Labor Market Interventions	265
Example 8.7	Treatment Effects on Earnings	266
Example 8.8	The Oregon Health Insurance Experiment	266
Example 8.9	The Effect of Counseling on Financial Management	266
Example 8.10	Treatment Effects on Earnings	271
Example 8.5	Labor Supply Model (Continued)	277
Example 8.11	Overidentification of the Labor Supply Equation	279
Example 8.12	Income and Education in a Study of Twins	286
Example 8.13	Instrumental Variables Estimates of the Consumption Function	291
Example 8.14	Does Television Watching Cause Autism?	292
Example 8.15	Is Season of Birth a Valid Instrument?	294

## **CHAPTER 9 The Generalized Regression Model and Heteroscedasticity 297**

Example 9.1	Heteroscedastic Regression and the White Estimator	300
Example 9.2	Testing for Heteroscedasticity	315
Example 9.3	Multiplicative Heteroscedasticity	315
Example 9.4	Groupwise Heteroscedasticity	318

## **CHAPTER 10 Systems of Regression Equations 326**

Example 10.1	A Regional Production Model for Public Capital	336
Example 10.2	Cobb–Douglas Cost Function	340
Example 10.3	A Cost Function for U.S. Manufacturing	344
Example 10.4.	Reverse Causality and Endogeneity in Health	347

## xxviii Examples and Applications

Example 10.5	Structure and Reduced Form in a Small Macroeconomic Model	351
Example 10.6	Identification of a Supply and Demand Model	355
Example 10.7	The Rank Condition and a Two-Equation Model	357
Example 10.8	Simultaneity in Health Production	360
Example 10.9	Klein's Model I	364

### **CHAPTER 11 Models for Panel Data 373**

Example 11.1	A Rotating Panel: The Survey of Income and Program Participation (SIPP) Data	378
Example 11.2	Attrition and Inverse Probability Weighting in a Model for Health	378
Example 11.3	Attrition and Sample Selection in an Earnings Model for Physicians	380
Example 11.4	Wage Equation	385
Example 11.5	Robust Estimators of the Wage Equation	389
Example 11.6	Analysis of Covariance and the World Health Organization (WHO) Data	392
Example 11.7	Fixed Effects Estimates of a Wage Equation	397
Example 11.8	Two-Way Fixed Effects with Unbalanced Panel Data	399
Example 11.9	Heterogeneity in Time Trends in an Aggregate Production Function	402
Example 11.10	Test for Random Effects	411
Example 11.11	Estimates of the Random Effects Model	412
Example 11.12	Hausman and Variable Addition Tests for Fixed versus Random Effects	416
Example 11.13	Hospital Costs	419
Example 11.14	Spatial Autocorrelation in Real Estate Sales	424
Example 11.15	Spatial Lags in Health Expenditures	426
Example 11.16	Endogenous Income in a Health Production Model	429
Example 11.17	The Returns to Schooling	432
Example 11.18	The Returns to Schooling	433
Example 11.19	Dynamic Labor Supply Equation	443
Example 11.20	Health Care Utilization	446
Example 11.21	Exponential Model with Fixed Effects	448
Example 11.22	Random Coefficients Model	452
Example 11.23	Fannie Mae's Pass Through	453
Example 11.24	Dynamic Panel Data Models	455
Example 11.25	A Mixed Fixed Growth Model for Developing Countries	459

### **CHAPTER 12 Estimation Frameworks in Econometrics 465**

Example 12.3	Joint Modeling of a Pair of Event Counts	472
Example 12.4	The Formula That Killed Wall Street 6	472
Example 12.5	Semiparametric Estimator for Binary Choice Models	475

Example 12.6	A Model of Vacation Expenditures	476
Example 12.1	The Linear Regression Model	468
Example 12.2	The Stochastic Frontier Model	468

### **CHAPTER 13 Minimum Distance Estimation and the Generalized Method of Moments 488**

Example 13.1	Euler Equations and Life Cycle Consumption	488
Example 13.2	Method of Moments Estimator for $N[\mu, \sigma^2]$	490
Example 13.3	Inverse Gaussian (Wald) Distribution	491
Example 13.4	Mixture of Normal Distributions	491
Example 13.5	Gamma Distribution	493
Example 13.5	(Continued)	495
Example 13.6	Minimum Distance Estimation of a Hospital Cost Function	498
Example 13.7	GMM Estimation of a Nonlinear Regression Model	504
Example 13.8	Empirical Moment Equation for Instrumental Variables	507
Example 13.9	Overidentifying Restrictions	511
Example 13.10	GMM Estimation of a Dynamic Panel Data Model of Local Government Expenditures	530

### **CHAPTER 14 Maximum Likelihood Estimation 537**

Example 14.1	Identification of Parameters	538
Example 14.2	Log-Likelihood Function and Likelihood Equations for the Normal Distribution	541
Example 14.3	Information Matrix for the Normal Distribution	548
Example 14.4	Variance Estimators for an MLE	550
Example 14.5	Two-Step ML Estimation	567
Example 14.6	A Regression with Nonnormal Disturbances	572
Example 14.7	Cluster Robust Standard Errors	574
Example 14.8	Logistic, t, and Skew Normal Disturbances	579
Example 14.9	Testing for Constant Returns to Scale	584
Example 14.10	Multiplicative Heteroscedasticity	589
Example 14.11	Maximum Likelihood Estimation of Gasoline Demand	590
Example 14.12	Identification in a Loglinear Regression Model	591
Example 14.13	Geometric Regression Model for Doctor Visits	597
Example 14.14	ML Estimates of a Seemingly Unrelated Regressions Model	602
Example 14.15	Maximum Likelihood and FGLS Estimates of a Wage Equation	608
Example 14.16	Statewide Productivity	610
Example 14.17	Random Effects Geometric Regression Model	617
Example 14.18	Fixed and Random Effects Geometric Regression	621
Example 14.19	A Normal Mixture Model for Grade Point Averages	623

**xxx** Examples and Applications

Example 14.20	Latent Class Regression Model for Grade Point Averages	625
Example 14.21	Predicting Class Probabilities	627
Example 14.22	A Latent Class Two-Part Model for Health Care Utilization	630
Example 14.23	Latent Class Models for Health Care Utilization	631
Example 14.24	Semiparametric Random Effects Model	634

**CHAPTER 15 Simulation-Based Estimation and Inference and Random Parameter Models 641**

Example 15.1	Inferring the Sampling Distribution of the Least Squares Estimator	641
Example 15.2	Bootstrapping the Variance of the LAD Estimator	641
Example 15.3	Least Simulated Sum of Squares	642
Example 15.4	Long-Run Elasticities	648
Example 15.5	Bootstrapping the Variance of the Median	651
Example 15.6	Block Bootstrapping Standard Errors and Confidence Intervals in a Panel	653
Example 15.7	Monte Carlo Study of the Mean Versus the Median	654
Example 15.8	Fractional Moments of the Truncated Normal Distribution	663
Example 15.9	Estimating the Lognormal Mean	666
Example 15.10	Poisson Regression Model with Random Effects	672
Example 15.11	Maximum Simulated Likelihood Estimation of the Random Effects Linear Regression Model	672
Example 15.12	Random Parameters Wage Equation	675
Example 15.13	Least Simulated Sum of Squares Estimates of a Production Function Model	677
Example 15.14	Hierarchical Linear Model of Home Prices	679
Example 15.15	Individual State Estimates of a Private Capital Coefficient	684
Example 15.16	Mixed Linear Model for Wages	685
Example 15.17	Maximum Simulated Likelihood Estimation of a Binary Choice Model	689

**CHAPTER 16 Bayesian Estimation and Inference 694**

Example 16.1	Bayesian Estimation of a Probability	696
Example 16.2	Estimation with a Conjugate Prior	701
Example 16.3	Bayesian Estimate of the Marginal Propensity to Consume	703
Example 16.4	Posterior Odds for the Classical Regression Model	706
Example 16.5	Gibbs Sampling from the Normal Distribution	708
Example 16.6	Gibbs Sampler for a Probit Model	712
Example 16.7	Bayesian and Classical Estimation of Heterogeneity in the Returns to Education	717



<b>CHAPTER 17 Binary Outcomes and Discrete Choices</b>			<b>725</b>
Example 17.1	Labor Force Participation Model	728	
Example 17.2	Structural Equations for a Binary Choice Model		730
Example 17.3	Probability Models	737	
Example 17.4	The Light Bulb Puzzle: Examining Partial Effects		739
Example 17.5	Cheating in the Chicago School System—An LPM		741
Example 17.6	Robust Covariance Matrices for Probit and LPM Estimators	745	
Example 17.7	Testing for Structural Break in a Logit Model		748
Example 17.8	Standard Errors for Partial Effects	752	
Example 17.9	Hypothesis Tests About Partial Effects	753	
Example 17.10	Confidence Intervals for Partial Effects	754	
Example 17.11	Inference About Odds Ratios	754	
Example 17.12	Interaction Effect	757	
Example 17.13	Prediction with a Probit Model	760	
Example 17.14	Fit Measures for a Logit Model	761	
Example 17.15	Specification Test in a Labor Force Participation Model	765	
Example 17.16	Distributional Assumptions	767	
Example 17.17	Credit Scoring	768	
Example 17.18	An Incentive Program for Quality Medical Care		771
Example 17.19	Moral Hazard in German Health Care	772	
Example 17.20	Labor Supply Model	776	
Example 17.21	Cardholder Status and Default Behavior	779	
Example 17.22	Binary Choice Models for Panel Data	789	
Example 17.23	Fixed Effects Logit Model: Magazine Prices Revisited		789
Example 17.24	Panel Data Random Effects Estimators	793	
Example 17.25	A Dynamic Model for Labor Force Participation and Disability	796	
Example 17.26	An Intertemporal Labor Force Participation Equation		796
Example 17.27	Semiparametric Models of Heterogeneity	797	
Example 17.28	Parameter Heterogeneity in a Binary Choice Model		799
Example 17.29	Nonresponse in the GSOEP Sample	802	
Example 17.30	A Spatial Logit Model for Auto Supplier Locations		806
Example 17.31	Tetrachoric Correlation	810	
Example 17.32	Bivariate Probit Model for Health Care Utilization		813
Example 17.33	Bivariate Random Effects Model for Doctor and Hospital Visits	814	
Example 17.34	The Impact of Catholic School Attendance on High School Performance	817	
Example 17.35	Gender Economics Courses at Liberal Arts Colleges		817
Example 17.36	A Multivariate Probit Model for Product Innovations		820
<b>CHAPTER 18 Multinomial Choices and Event Counts</b>			<b>826</b>
Example 18.1	Hollingshead Scale of Occupations	831	
Example 18.2	Home Heating Systems	832	

**xxxii** Examples and Applications

Example 18.3	Multinomial Choice Model for Travel Mode	839
Example 18.4	Using Mixed Logit to Evaluate a Rebate Program	847
Example 18.5	Latent Class Analysis of the Demand for Green Energy	849
Example 18.6	Malaria Control During Pregnancy	852
Example 18.7	Willingness to Pay for Renewable Energy	855
Example 18.8	Stated Choice Experiment: Preference for Electricity Supplier	860
Example 18.9	Health Insurance Market	865
Example 18.10	Movie Ratings	867
Example 18.11	Rating Assignments	870
Example 18.12	Brant Test for an Ordered Probit Model of Health Satisfaction	873
Example 18.13	Calculus and Intermediate Economics Courses	873
Example 18.14	Health Satisfaction	877
Example 18.15	A Dynamic Ordered Choice Model:	878
Example 18.16	Count Data Models for Doctor Visits	892
Example 18.17	Major Derogatory Reports	896
Example 18.18	Extramarital Affairs	897
Example 18.19	Panel Data Models for Doctor Visits	904
Example 18.20	Zero-Inflation Models for Major Derogatory Reports	906
Example 18.21	Hurdle Models for Doctor Visits	909
Example 18.22	Endogenous Treatment in Health Care Utilization	913

**CHAPTER 19 Limited Dependent Variables—Truncation, Censoring, and Sample Selection 918**

Example 19.1	Truncated Uniform Distribution	920
Example 19.2	A Truncated Lognormal Income Distribution	921
Example 19.3	Stochastic Cost Frontier for Swiss Railroads	928
Example 19.4	Censored Random Variable	933
Example 19.5	Estimated Tobit Equations for Hours Worked	937
Example 19.6	Two-Part Model For Extramarital Affairs	942
Example 19.7	Multiplicative Heteroscedasticity in the Tobit Model	946
Example 19.8	Incidental Truncation	949
Example 19.9	A Model of Labor Supply	950
Example 19.10	Female Labor Supply	956
Example 19.11	A Mover-Stayer Model for Migration	957
Example 19.12	Doctor Visits and Insurance	958
Example 19.13	Survival Models for Strike Duration	975
Example 19.14	Time Until Retirement	976

**CHAPTER 20 Serial Correlation 981**

Example 20.1	Money Demand Equation	981
Example 20.2	Autocorrelation Induced by Misspecification of the Model	982

Example 20.3	Negative Autocorrelation in the Phillips Curve	983
Example 20.4	Autocorrelation Function for the Rate of Inflation	988
Example 20.5	Autocorrelation Consistent Covariance Estimation	999
Example 20.6	Test for Autocorrelation	1001
Example 20.7	Dynamically Complete Regression	1009
Example 20.8	Stochastic Volatility	1011
Example 20.9	GARCH Model for Exchange Rate Volatility	1017

## **CHAPTER 21 Nonstationary Data 1022**

Example 21.1	A Nonstationary Series	1024
Example 21.2	Tests for Unit Roots	1030
Example 21.3	Augmented Dickey–Fuller Test for a Unit Root in GDP	1037
Example 21.4	Is there a Unit Root in GDP?	1039
Example 21.5	Cointegration in Consumption and Output	1040
Example 21.6	Several Cointegrated Series	1041
Example 21.7	Multiple Cointegrating Vectors	1043
Example 21.8	Cointegration in Consumption and Output	1046

## **Online Appendix C Estimation and Inference C-1**

Example C.1	Descriptive Statistics for a Random Sample	C-4
Example C.2	Kernel Density Estimator for the Income Data	C-5
Example C.3	Sampling Distribution of A Sample Mean	C-7
Example C.4	Sampling Distribution of the Sample Minimum	C-7
Example C.5	Mean Squared Error of The Sample Variance	C-11
Example C.6	Likelihood Functions for Exponential and Normal Distributions	C-12
Example C.7	Variance Bound for the Poisson Distribution	C-13
Example C.8	Confidence Intervals for the Normal Mean	C-14
Example C.9	Estimated Confidence Intervals for a Normal Mean and Variance	C-15
Example C.10	Testing a Hypothesis About a Mean	C-17
Example C.11	Consistent Test About a Mean	C-19
Example C.12	Testing A Hypothesis About a Mean with a Confidence Interval	C-19
Example C.13	One-Sided Test About a Mean	D-1

## **Online Appendix D Large-Sample Distribution Theory D-1**

Example D.1	Mean Square Convergence of the Sample Minimum in Exponential Sampling	D-4
Example D.2	Estimating a Function of the Mean	D-5
Example D.3	Probability Limit of a Function of $\bar{x}$ and $s^2$	D-9
Example D.4	Limiting Distribution of $t_{n-2}$	D-12
Example D.5	The F Distribution	D-14
Example D.6	The Lindeberg–Levy Central Limit Theorem	D-16

**xxxiv** Examples and Applications

- Example D.7 Asymptotic Distribution of the Mean of an Exponential  
Sample D-20
- Example D.8 Asymptotic Inefficiency of the Median In Normal  
Sampling D-21
- Example D.9 Asymptotic Distribution of a Function of Two  
Estimators D-22
- Example D.10 Asymptotic Moments of the Normal Sample  
Variance D-23

## PREFACE



### ECONOMETRIC ANALYSIS

*Econometric Analysis* is a broad introduction to the field of econometrics. This field grows continually. A (not complete) list of journals devoted at least in part to econometrics now includes: *Econometric Reviews*; *Econometric Theory*; *Econometrica*; *Econometrics*; *Econometrics and Statistics*; *The Econometrics Journal*; *Empirical Economics*; *Foundations and Trends in Econometrics*; *The Journal of Applied Econometrics*; *The Journal of Business and Economic Statistics*; *The Journal of Choice Modelling*; *The Journal of Econometric Methods*; *The Journal of Econometrics*; *The Journal of Time Series Analysis*; *The Review of Economics and Statistics*. Constructing a textbook-style survey to introduce the topic at a graduate level has become increasingly ambitious. Nonetheless, that is what I seek to do here. This text attempts to present, at an entry graduate level, enough of the topics in econometrics that a student can comfortably move on from here to practice or to more advanced study. For example, the literature on “Treatment Effects” is already vast, rapidly growing, complex in the extreme, and occasionally even contradictory. But, there are a few bedrock principles presented in Chapter 8 that (I hope) can help the interested practitioner or student get started as they wade into this segment of the literature. The book is intended as a bridge between an introduction to econometrics and the professional literature.

The book has two objectives. The first is to introduce students to *applied econometrics*, including basic techniques in linear regression analysis and some of the rich variety of models that are used when the linear model proves inadequate or inappropriate. Modern software has made complicated modeling very easy to put into practice. The second objective is to present sufficient *theoretical background* so that the reader will (1) understand the advanced techniques that are made so simple in modern software and (2) recognize new variants of the models learned about here as merely natural extensions that fit within a common body of principles. This book contains a substantial amount of theoretical material, such as that on the GMM, maximum likelihood estimation, and asymptotic results for regression models.

One overriding purpose has motivated all eight editions of *Econometric Analysis*. The vast majority of readers of this book will be users, not developers, of econometrics. I believe that it is not sufficient to teach econometrics by reciting (and proving) the theories of estimation and inference. Although the often-subtle theory is extremely important, the application is equally crucial. To that end, I have provided hundreds of worked numerical examples and extracts from applications in the received empirical literature in many fields. My purpose in writing this work, and in my continuing efforts to update it, is to show readers how to *do* econometric analysis. But, I also believe that readers want (and need) to know what is going on behind the curtain when they use ever more sophisticated modern software for ever more complex econometric analyses.

I have taught econometrics at the level of *Econometric Analysis* at NYU for many years. I ask my students to learn how to use a (any) modern econometrics program as part of their study. I've lost track of the number of my students who recount to me their disappointment in a previous course in which they were taught how to use software, but not the theory and motivation of the techniques. In October, 2014, Google Scholar published its list of the 100 most cited works over all fields and all time. ([www.nature.com/polopoly\\_fs/721245!/file/GoogleScholartop100.xlsx](http://www.nature.com/polopoly_fs/721245!/file/GoogleScholartop100.xlsx)). *Econometric Analysis*, the only work in econometrics on the list, ranked number 34 with 48,100 citations. (As of this writing, November 2016, the number of citations to the first 7 editions in all languages approaches 60,000.) I take this extremely gratifying result as evidence that there are readers in many fields who agree that the practice of econometrics calls for an understanding of *why*, as well as *how* to use the tools in modern software. This book is for them.

## THE EIGHTH EDITION OF *ECONOMETRIC ANALYSIS*

This text is intended for a one-year graduate course for social scientists. Prerequisites should include calculus, mathematical statistics, and an introduction to econometrics at the level of, say, Gujarati and Porter's (2011) *Basic Econometrics*, Stock and Watson's (2014) *Introduction to Econometrics*, Kennedy's (2008) *Guide to Econometrics*, or Wooldridge's (2015) *Introductory Econometrics: A Modern Approach*. I assume, for example, that the reader has already learned about the basics of econometric methodology including the fundamental role of economic and statistical assumptions; the distinctions between cross-section, time-series, and panel data sets; and the essential ingredients of estimation, inference, and prediction with the multiple linear regression model. Self-contained (for our purposes) summaries of the matrix algebra, mathematical statistics, and statistical theory used throughout the book are given in Appendices A through D. I rely heavily on matrix algebra throughout. This may be a bit daunting to some early on but matrix algebra is an indispensable tool and I hope the reader will come to agree that it is a means to an end, not an end in itself. With matrices, the unity of a variety of results will emerge without being obscured by a curtain of summation signs. Appendix E and Chapter 15 contain a description of numerical methods that will be useful to practicing econometricians (and to us in the later chapters of the book).

Estimation of advanced nonlinear models is now as routine as least squares. I have included five chapters on estimation methods used in current research and five chapters on applications in micro- and macroeconometrics. The nonlinear models used in these fields are now the staples of the applied econometrics literature. As a consequence, this book also contains a fair amount of material that will extend beyond many first courses in econometrics. Once again, I have included this in the hope of laying a foundation for study of the professional literature in these areas.

## PLAN OF THE BOOK

The arrangement of the book is as follows:

Part I begins the formal development of econometrics with its fundamental pillar, the *linear multiple regression model*. Estimation and inference with the linear least squares estimator are analyzed in Chapters 2 through 6. The *nonlinear regression model* is introduced

in Chapter 7 along with quantile, semi- and nonparametric regression, all as extensions of the familiar linear model. *Instrumental variables estimation* is developed in Chapter 8.

Part II presents three major extensions of the regression model. Chapter 9 presents the consequences of relaxing one of the main assumptions of the linear model, homoscedastic nonautocorrelated disturbances, to introduce the *generalized regression model*. The focus here is on heteroscedasticity; autocorrelation is mentioned, but a detailed treatment is deferred to Chapter 20 in the context of time-series data. Chapter 10 introduces systems of regression equations, in principle, as the approach to modeling simultaneously a set of random variables and, in practical terms, as an extension of the generalized linear regression model. Finally, *panel data methods*, primarily fixed and random effects models of heterogeneity, are presented in Chapter 11.

The second half of the book is devoted to topics that extend the linear regression model in many directions. Beginning with Chapter 12, we proceed to the more involved methods of analysis that contemporary researchers use in analysis of “real-world” data. Chapters 12 to 16 in Part III present different estimation methodologies. Chapter 12 presents an overview by making the distinctions between *parametric*, *semiparametric* and *nonparametric methods*. The leading application of semiparametric estimation in the current literature is the *generalized method of moments (GMM) estimator* presented in Chapter 13. This technique provides the platform for much of modern econometrics. *Maximum likelihood estimation* is developed in Chapter 14. *Monte Carlo* and *simulation-based methods* such as *bootstrapping* that have become a major component of current research are developed in Chapter 15. Finally, *Bayesian methods* are introduced in Chapter 16.

Parts IV and V develop two major subfields of econometric methods, *microeconometrics*, which is typically based on cross-section and panel data, and *macroeconometrics*, which is usually associated with analysis of time-series data. In Part IV, Chapters 17 to 19 are concerned with models of *discrete choice*, *censoring*, *truncation*, *sample selection*, *duration* and the analysis of *counts of events*. In Part V, Chapters 20 and 21, we consider two topics in time-series analysis, models of *serial correlation* and regression models for *nonstationary data*—the usual substance of macroeconomic analysis.

## REVISIONS

With only a couple exceptions noted below, I have retained the broad outline of the text. I have revised the presentation throughout the book (including this preface) to streamline the development of topics, in some cases (I hope), to improve the clarity of the derivations. Major revisions include:

- I have moved the material related to “causal inference” forward to the early chapters of the book – these topics are now taught earlier in the graduate sequence than heretofore and I’ve placed them in the context of the models and methods where they appear rather than as separate topics in the more advanced sections of the seventh edition. Difference in difference regression as a method, and regression discontinuity designs now appear in Chapter 6 with the discussion of functional forms and in the context of extensive applications extracted from the literature. The analysis of treatment effects has all been moved from Chapter 19 (on censoring and truncation) to Chapter 8 on endogeneity under the heading of “Endogenous

Dummy Variables.” Chapter 8, as a whole, now includes a much more detailed treatment of instrumental variable methods.

- I have added many new examples, some as extracts from applications in the received literature, and others as worked numerical examples. I have drawn applications from many different fields including industrial organization, transportation, health economics, popular culture and sports, urban development and labor economics.
- Chapter 10 on systems of equations has been shifted (yet further) from its early emphasis on formal simultaneous linear equations models to systems of regression equations and the leading application, the single endogenous variable in a two equation recursive model – this is the implicit form of the regression model that contains one “endogenous” variable.
- The use of robust estimation and inference methods has been woven more extensively into the general methodology, in practice and throughout this text. The ideas of robust estimation and inference are introduced immediately with the linear regression model in Chapters 4 and 5, rather than as accommodations to nonspherical disturbances in Chapter 9. The role that a robust variance estimator will play in the Wald statistic is developed immediately when the result is first presented in Chapter 5.
- Chapters 4 (Least Squares), 6 (Functional Forms), 8 (Endogeneity), 10 (Equation Systems) and 11 (Panel Data) have been heavily revised to emphasize both contemporary econometric methods and the applications.
- I have moved Appendices A-F to the Companion Web site, at [www.pearsonhighered.com/greene](http://www.pearsonhighered.com/greene), that accompanies this text. Students can access them at no cost.

The first semester of study in a course based on *Econometric Analysis* would focus on Chapters 1-6 (the linear regression model), 8 (endogeneity and causal modeling), and possibly some of 11 (panel data). Most of the revisions in the eighth edition appear in these chapters.

## SOFTWARE AND DATA

There are many computer programs that are widely used for the computations described in this book. All were written by econometricians or statisticians, and in general, all are regularly updated to incorporate new developments in applied econometrics. A sampling of the most widely used packages and Web sites where you can find information about them are

EViews	<a href="http://www.eviews.com">www.eviews.com</a>	(QMS, Irvine, CA)
Gauss	<a href="http://www.aptech.com">www.aptech.com</a>	(Aptech Systems, Kent, WA)
LIMDEP	<a href="http://www.limdep.com">www.limdep.com</a>	(Econometric Software, Plainview, NY)
MATLAB	<a href="http://www.mathworks.com">www.mathworks.com</a>	(Mathworks, Natick, MA)
NLOGIT	<a href="http://www.nlogit.com">www.nlogit.com</a>	(Econometric Software, Plainview, NY)
R	<a href="http://www.r-project.org/">www.r-project.org/</a>	(The R Project for Statistical Computing)
RATS	<a href="http://www.estima.com">www.estima.com</a>	(Estima, Evanston, IL)
SAS	<a href="http://www.sas.com">www.sas.com</a>	(SAS, Cary, NC)
Shazam	<a href="http://econometrics.com">econometrics.com</a>	(Northwest Econometrics Ltd., Gibsons, Canada)
Stata	<a href="http://www.stata.com">www.stata.com</a>	(Stata, College Station, TX)



A more extensive list of computer software used for econometric analysis can be found at the resource Web site, <http://www.oswego.edu/~economic/econsoftware.htm>.

With only a few exceptions, the computations described in this book can be carried out with any of the packages listed. *NLOGIT* was used for the computations in most of the applications. This text contains no instruction on using any particular program or language. Many authors have produced *RATS*, *LIMDEP/NLOGIT*, *EViews*, *SAS*, or *Stata* code for some of the applications, including, in a few cases, in the documentation for their computer programs. There are also quite a few volumes now specifically devoted to econometrics associated with particular packages, such as Cameron and Trivedi's (2009) companion to their treatise on microeconometrics.

The data sets used in the examples are also available on the Web site for the text, <http://people.stern.nyu.edu/wgreene/Text/econometricanalysis.htm>. Throughout the text, these data sets are referred to "Table Fn.m," for example Table F4.1. The "F" refers to Appendix F available on the Companion web site which contains descriptions of the data sets. The actual data are posted in generic ASCII and portable formats on the Web site with the other supplementary materials for the text. There are now thousands of interesting Web sites containing software, data sets, papers, and commentary on econometrics. It would be hopeless to attempt any kind of a survey. One code/data site that is particularly agreeably structured and well targeted for readers of this book is the data archive for the *Journal of Applied Econometrics (JAE)*. They have archived all the nonconfidential data sets used in their publications since 1988 (with some gaps before 1995). This useful site can be found at <http://qed.econ.queensu.ca/jae/>. Several of the examples in the text use the *JAE* data sets. Where we have done so, we direct the reader to the *JAE*'s Web site, rather than our own, for replication. Other journals have begun to ask their authors to provide code and data to encourage replication. Another easy-to-navigate site for aggregate data on the U.S. economy is <https://datahub.io/dataset/economagic>.

## ACKNOWLEDGMENTS

It is a pleasure to express my appreciation to those who have influenced this work. I remain grateful to Arthur Goldberger (dec.), Arnold Zellner (dec.), Dennis Aigner, Bill Becker, and Laurits Christensen for their encouragement and guidance. After eight editions of this book, the number of individuals who have significantly improved it through their comments, criticisms, and encouragement has become far too large for me to thank each of them individually. I am grateful for their help and I hope that all of them see their contribution to this edition. Any number of people have submitted tips about the text. You can find many of them listed in the errata pages on the text Web site, <http://people.stern.nyu.edu/wgreene/Text/econometricanalysis.htm>, in particular: David Hoaglin, University of Massachusetts; Randall Campbell, Mississippi State University; Carter Hill, Louisiana State University; and Tom Doan, Estima Corp. I would also like to thank two colleagues who have worked on translations of *Econometric Analysis*, Marina Turuntseva (the Russian edition) and Umit Senesen (the Turkish translation). I must also acknowledge the mail I've received from hundreds of readers and practitioners from the world over who have given me a view into topics and questions that practitioners are interested in, and have provided a vast trove of helpful material for my econometrics courses.

I also acknowledge the many reviewers of my work whose careful reading has vastly improved the book through this edition: Scott Atkinson, University of Georgia; Badi Baltagi, Syracuse University; Neal Beck, New York University; William E. Becker (Ret.), Indiana University; Eric J. Belasko, Texas Tech University; Anil Bera, University of Illinois; John Burkett, University of Rhode Island; Leonard Carlson, Emory University; Frank Chaloupka, University of Illinois at Chicago; Chris Cornwell, University of Georgia; Craig Depken II, University of Texas at Arlington; Frank Diebold, University of Pennsylvania; Edward Dwyer, Clemson University; Michael Ellis, Wesleyan University; Martin Evans, Georgetown University; Vahagn Galstyan, Trinity College Dublin; Paul Glewwe, University of Minnesota; Ed Greenberg, Washington University at St. Louis; Miguel Herce, University of North Carolina; Joseph Hilbe, Arizona State University; Dr. Uwe Jensen, Christian-Albrecht University; K. Rao Kadiyala, Purdue University; William Lott, University of Connecticut; Thomas L. Marsh, Washington State University; Edward Mathis, Villanova University; Mary McGarvey, University of Nebraska–Lincoln; Ed Melnick, New York University; Thad Mirer, State University of New York at Albany; Cyril Pasche, University of Geneva; Paul Ruud, University of California at Berkeley; Sherrie Rhine, Federal Deposit Insurance Corp.; Terry G. Seaks (Ret.), University of North Carolina at Greensboro; Donald Snyder, California State University at Los Angeles; Steven Stern, University of Virginia; Houston Stokes, University of Illinois at Chicago; Dmitrios Thomakos, Columbia University; Paul Wachtel, New York University; Mary Beth Walker, Georgia State University; Mark Watson, Harvard University; and Kenneth West, University of Wisconsin. My numerous discussions with Bruce McCullough of Drexel University have improved Appendix E and at the same time increased my appreciation for numerical analysis. I am especially grateful to Jan Kiviet of the University of Amsterdam, who subjected my third edition to a microscopic examination and provided literally scores of suggestions, virtually all of which appear herein. Professor Pedro Bacao, University of Coimbra, Portugal, and Mark Strahan of Sand Hill Econometrics and Umit Senesen of Istanbul Technical University did likewise with the sixth and seventh editions.

I would also like to thank the many people at Pearson Education who have put this book together with me: Adrienne D' Ambrosio, Neeraj Bhalla, Sugandh Juneja, and Nicole Suddeth and the composition team at SPi Global.

For over 25 years since the first edition, I've enjoyed the generous support and encouragement of many people, some close to me, especially my family, and many not so close. I'm especially grateful for the help, support and priceless encouragement of my wife, Sherrie Rhine, whose unending enthusiasm for this project has made it much less daunting, and much more fun.

William H. Greene  
February 2017

# ECONOMETRICS



## 1.1 INTRODUCTION

This book will present an introductory survey of econometrics. We will discuss the fundamental ideas that define the methodology and examine a large number of specific models, tools, and methods that econometricians use in analyzing data. This chapter will introduce the central ideas that are the paradigm of econometrics. Section 1.2 defines the field and notes the role that theory plays in motivating econometric practice. Sections 1.3 and 1.4 discuss the types of applications that are the focus of econometric analyses. The process of econometric modeling is presented in Section 1.5 with a classic application, Keynes's consumption function. A broad outline of the text is presented in Section 1.6. Section 1.7 notes some specific aspects of the presentation, including the use of numerical examples and the mathematical notation that will be used throughout the text.

## 1.2 THE PARADIGM OF ECONOMETRICS

In the first issue of *Econometrica*, the Econometric Society stated that its main object shall be to promote studies that aim at a unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking similar to that which has come to dominate the natural sciences. . . . But there are several aspects of the quantitative approach to economics, and no single one of these aspects taken by itself, should be confounded with econometrics. Thus, econometrics is by no means the same as economic statistics. Nor is it identical with what we call general economic theory, although a considerable portion of this theory has a definitely quantitative character. Nor should econometrics be taken as synonymous [sic] with the application of mathematics to economics. Experience has shown that each of these three viewpoints, that of statistics, economic theory, and mathematics, is a necessary, but not by itself a sufficient, condition for a real understanding of the quantitative relations in modern economic life. It is the *unification* of all three that is powerful. And it is this unification that constitutes econometrics.

The Society responded to an unprecedented accumulation of statistical information. It saw a need to establish a body of principles that could organize what would otherwise become a bewildering mass of data. Neither the pillars nor the objectives of econometrics have changed in the years since this editorial appeared. Econometrics concerns itself with the

application of mathematical statistics and the tools of statistical inference to the empirical measurement of relationships postulated by an underlying theory.

It is interesting to observe the response to a contemporary, likewise unprecedented accumulation of massive amounts of quantitative information in the form of “Big Data.” Consider the following assessment of what Kitchin (2014) sees as a paradigm shift in the analysis of data.

This article examines how the availability of Big Data, coupled with new data analytics, challenges established epistemologies across the sciences, social sciences and humanities, and assesses the extent to which they are engendering paradigm shifts across multiple disciplines. In particular, it critically explores new forms of empiricism that declare ‘the end of theory,’ the creation of data-driven rather than knowledge-driven science, and the development of digital humanities and computational social sciences that propose radically different ways to make sense of culture, history, economy and society. It is argued that: (1) Big Data and new data analytics are disruptive innovations which are reconfiguring in many instances how research is conducted; and (2) there is an urgent need for wider critical reflection within the academy on the epistemological implications of the unfolding data revolution, a task that has barely begun to be tackled despite the rapid changes in research practices presently taking place.

We note the suggestion that data-driven analytics are proposed to replace theory (and econometrics as envisioned by Frisch) for providing the organizing principles to guide empirical research. (We will examine an example in Chapter 18 where we consider analyzing survey data with ordered choice models. Also, see Varian (2014) for a more balanced view.) The focus is driven partly by the startling computational power that would have been unavailable to Frisch. It seems likely that the success of this new paradigm will turn at least partly on the questions pursued. Whether the interesting features of an underlying data-generating process can be revealed by appealing to the data themselves without a theoretical platform seems to be a prospect raised by the author. The article does focus on the role of an underlying theory in empirical research—this is a central pillar of econometric methodology. As of this writing, the success story of Big Data analysis is still being written.

The crucial role that econometrics plays in economics has grown over time. The Nobel Prize in Economics has recognized this contribution with numerous awards to econometricians, including the first which was given to (the same) Ragnar Frisch in 1969. Lawrence Klein in 1980, Trygve Haavelmo in 1989, James Heckman and Daniel McFadden in 2000, Robert Engle and Clive Granger in 2003. Christopher Sims in 2011 and Lars Hansen in 2013 were recognized for their empirical research. The 2000 prize was noteworthy in that it celebrated the work of two scientists whose research was devoted to the marriage of behavioral theory and econometric modeling.

### **Example 1.1 Behavioral Models and the Nobel Laureates**

The pioneering work by both James Heckman and Dan McFadden rests firmly on a theoretical foundation of utility maximization.

For Heckman’s, we begin with the standard theory of household utility maximization over consumption and leisure. The textbook model of utility maximization produces a demand for leisure time that translates into a supply function of labor. When home production (i.e., work

in the home as opposed to the outside, formal labor market) is considered in the calculus, then desired *hours* of (formal) labor can be negative. An important conditioning variable is the *reservation wage*—the wage rate that will induce formal labor market participation. On the demand side of the labor market, we have firms that offer market wages that respond to such attributes as age, education, and experience. What can we learn about labor supply behavior based on observed market wages, these attributes, and observed hours in the formal market? Less than it might seem, intuitively because our observed data omit half the market—the data on formal labor market activity are not randomly drawn from the whole population.

Heckman's observations about this implicit truncation of the distribution of hours or wages revolutionized the analysis of labor markets. Parallel interpretations have since guided analyses in every area of the social sciences. The analysis of policy interventions such as education initiatives, job training and employment policies, health insurance programs, market creation, financial regulation, and a host of others is heavily influenced by Heckman's pioneering idea that when participation is part of the behavior being studied, the analyst must be cognizant of the impact of common influences in both the presence of the intervention and the outcome. We will visit the literature on sample selection and treatment/program evaluation in Chapters 5, 6, 8 and 19.

Textbook presentations of the theories of demand for goods that produce utility, because they deal in continuous variables, are conspicuously silent on the kinds of discrete choices that consumers make every day—what brand of product to choose, whether to buy a large commodity such as a car or a refrigerator, how to travel to work, whether to rent or buy a home, where to live, what candidate to vote for, and so on. Nonetheless, a model of *random utility* defined over the alternatives available to the consumer provides a theoretically sound platform for studying such choices. Important variables include, as always, income and relative prices. What can we learn about underlying preference structures from the discrete choices that consumers make? What must be assumed about these preferences to allow this kind of inference? What kinds of statistical models will allow us to draw inferences about preferences? McFadden's work on how commuters choose to travel to work, and on the underlying theory appropriate to this kind of modeling, has guided empirical research in discrete consumer choices for several decades. We will examine McFadden's models of discrete choice in Chapter 18.

### 1.3 THE PRACTICE OF ECONOMETRICS

We can make a useful distinction between *theoretical econometrics* and *applied econometrics*. Theorists develop new techniques for estimation and hypothesis testing and analyze the consequences of applying particular methods when the assumptions that justify those methods are not met. Applied econometricians are the users of these techniques and the analysts of data (real world and simulated). The distinction is far from sharp; practitioners routinely develop new analytical tools for the purposes of the study that they are involved in. This text contains a large amount of econometric theory, but it is directed toward applied econometrics. We have attempted to survey techniques, admittedly some quite elaborate and intricate, that have seen wide use in the field.

Applied econometric methods will be used for estimation of important quantities, analysis of economic outcomes such as policy changes, markets or individual behavior, testing theories, and for forecasting. The last of these is an art and science in itself that is the subject of a vast library of sources. Although we will briefly discuss some

aspects of forecasting, our interest in this text will be on estimation and analysis of models. The presentation, where there is a distinction to be made, will contain a blend of microeconomic and macroeconomic techniques and applications. It is also necessary to distinguish between *time-series analysis* (which is not our focus) and methods that primarily use *time-series data*. The former is, like forecasting, a growth industry served by its own literature in many fields. While we will employ some of the techniques of time-series analysis, we will spend relatively little time developing first principles.

## 1.4 MICROECONOMETRICS AND MACROECONOMETRICS

The connection between underlying behavioral models and the modern practice of econometrics is increasingly strong. Another distinction is made between *microeconomics* and *macroeconomics*. The former is characterized by its analysis of cross section and panel data and by its focus on individual consumers, firms, and micro-level decision makers. Practitioners rely heavily on the theoretical tools of microeconomics including utility maximization, profit maximization, and market equilibrium. The analyses are directed at subtle, difficult questions that often require intricate formulations. A few applications are as follows:

- What are the likely effects on labor supply behavior of proposed negative income taxes? [Ashenfelter and Heckman (1974)]
- Does attending an elite college bring an expected payoff in expected lifetime income sufficient to justify the higher tuition? [Kreuger and Dale (1999) and Kreuger (2000)]
- Does a voluntary training program produce tangible benefits? Can these benefits be accurately measured? [Angrist (2001)]
- Does an increase in the minimum wage lead to reduced employment? [Card and Krueger (1994)]
- Do smaller class sizes bring real benefits in student performance? [Hanushek (1999), Hoxby (2000), and Angrist and Lavy (1999)]
- Does the presence of health insurance induce individuals to make heavier use of the health care system—is moral hazard a measurable problem? [Riphahn et al. (2003)]
- Did the intervention addressing anticompetitive behavior of a group of 50 boarding schools by the UK Office of Fair Trading produce a measurable impact on fees charged? [Pesaresi, Flanagan, Scott, and Tragear (2015)]

Macroeconomics is involved in the analysis of time-series data, usually of broad aggregates such as price levels, the money supply, exchange rates, output, investment, economic growth, and so on. The boundaries are not sharp. For example, an application that we will examine in this text concerns spending patterns of municipalities, which rests somewhere between the two fields. The very large field of financial econometrics is concerned with long time-series data and occasionally vast panel data sets, but with a sharply focused orientation toward models of individual behavior. The analysis of market returns and exchange rate behavior is neither exclusively macro- nor microeconomic. [We will not be spending any time in this text on financial econometrics. For those with an interest in this field, We would recommend the celebrated work by Campbell, Lo, and Mackinlay (1997), or for a more time-series-oriented approach, Tsay (2005).]

Macroeconomic model builders rely on the interactions between economic agents and policy makers. For example:

- Does a monetary policy regime that is strongly oriented toward controlling inflation impose a real cost in terms of lost output on the U.S. economy? [Cecchetti and Rich (2001)]
- Did 2001's largest federal tax cut in U.S. history contribute to or dampen the concurrent recession? Or was it irrelevant?

Each of these analyses would depart from a formal model of the process underlying the observed data.

The techniques used in econometrics have been employed in a widening variety of fields, including political methodology, sociology,<sup>1</sup> health economics, medical research (e.g., how do we handle attrition from medical treatment studies?) environmental economics, economic geography, transportation engineering, and numerous others. Practitioners in these fields and many more are all heavy users of the techniques described in this text.

## 1.5 ECONOMETRIC MODELING

Econometric analysis usually begins with a statement of a theoretical proposition. Consider, for example, a classic application by one of Frisch's contemporaries:

### **Example 1.2** *Keynes's Consumption Function*

From Keynes's (1936) *General Theory of Employment, Interest and Money*:

We shall therefore define what we shall call the propensity to consume as the functional relationship  $f$  between  $X$ , a given level of income, and  $C$ , the expenditure on consumption out of the level of income, so that  $C = f(X)$ .

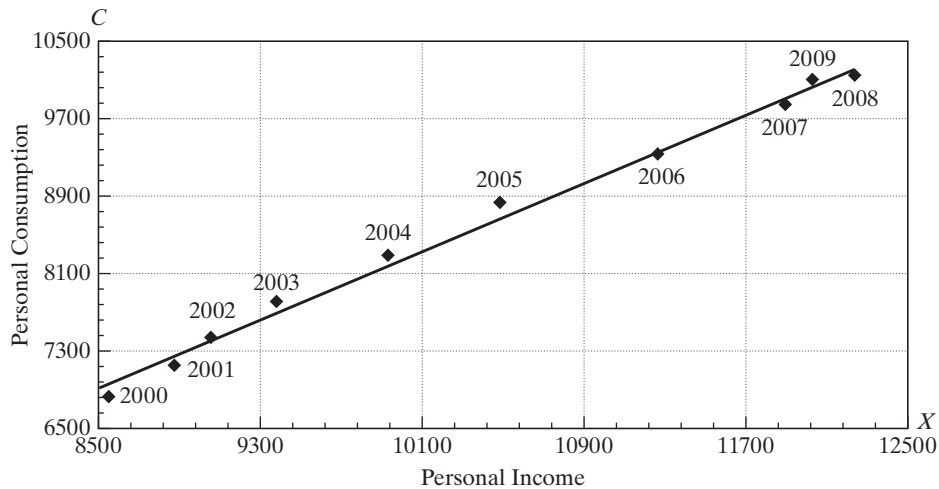
The amount that the community spends on consumption depends (i) partly on the amount of its income, (ii) partly on other objective attendant circumstances, and (iii) partly on the subjective needs and the psychological propensities and habits of the individuals composing it. The fundamental psychological law upon which we are entitled to depend with great confidence, both a priori from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income. That is, . . .  $dC/dX$  is positive and less than unity.

But, apart from short period changes in the level of income, it is also obvious that a higher absolute level of income will tend as a rule to widen the gap between income and consumption. . . . These reasons will lead, as a rule, to a greater proportion of income being saved as real income increases.

The theory asserts a relationship between consumption and income,  $C = f(X)$ , and claims in the second paragraph that the marginal propensity to consume (MPC),  $dC/dX$ , is between zero and one.<sup>2</sup> The final paragraph asserts that the average propensity to consume (APC),  $C/X$ , falls as income rises, or  $d(C/X)/dX = (MPC - APC)/X < 0$ . It follows that  $MPC < APC$ . The

<sup>1</sup> See, for example, Long (1997) and DeMaris (2004).

<sup>2</sup> Modern economists are rarely this confident about their theories. More contemporary applications generally begin from first principles and behavioral axioms, rather than simple observation.

**FIGURE 1.1** Aggregate U.S. Consumption and Income Data, 2000–2009.

most common formulation of the consumption function is a linear relationship,  $C = \alpha + X\beta$ , that satisfies Keynes's "laws" if  $\beta$  lies between zero and one and if  $\alpha$  is greater than zero.

These theoretical propositions provide the basis for an econometric study. Given an appropriate data set, we could investigate whether the theory appears to be consistent with the observed "facts." For example, we could see whether the linear specification appears to be a satisfactory description of the relationship between consumption and income, and, if so, whether  $\alpha$  is positive and  $\beta$  is between zero and one. Some issues that might be studied are (1) whether this relationship is stable through time or whether the parameters of the relationship change from one generation to the next (a change in the average propensity to save,  $1 - APC$ , might represent a fundamental change in the behavior of consumers in the economy); (2) whether there are systematic differences in the relationship across different countries, and, if so, what explains these differences; and (3) whether there are other factors that would improve the ability of the model to explain the relationship between consumption and income. For example, Figure 1.1 presents aggregate consumption and personal income in constant dollars for the United States for the 10 years of 2000–2009. (See Appendix Table F1.1.) Apparently, at least superficially, the data (the facts) are consistent with the theory. The relationship appears to be linear, albeit only approximately, the intercept of a line that lies close to most of the points is positive and the slope is less than one, although not by much. (However, if the line is fit by linear least squares regression, the intercept is negative, not positive.) Moreover, observers might disagree on what is meant by relationship in this description.

Economic theories such as Keynes's are typically sharp and unambiguous. Models of demand, production, labor supply, individual choice, educational attainment, income and wages, investment, market equilibrium, and aggregate consumption all specify precise, *deterministic relationships*. Dependent and independent variables are identified, a functional form is specified, and in most cases, at least a qualitative statement is made about the directions of effects that occur when independent variables in the model change. The model is only a simplification of reality. It will include the salient features of the relationship of interest but will leave unaccounted for influences that might well be present but are regarded as unimportant.



Correlations among economic variables are easily observable through descriptive statistics and techniques such as linear regression methods. The ultimate goal of the econometric model builder is often to uncover the deeper causal connections through elaborate structural, behavioral models. Note, for example, Keynes's use of the behavior of a *representative consumer* to motivate the behavior of macroeconomic variables, such as income and consumption. Heckman's model of labor supply noted in Example 1.1 is framed in a model of individual behavior. Berry, Levinsohn, and Pakes's (1995) detailed model of equilibrium pricing in the automobile market is another.

No model could hope to encompass the myriad essentially random aspects of economic life. It is thus also necessary to incorporate stochastic elements. As a consequence, observations on a variable will display variation attributable not only to differences in variables that are explicitly accounted for in the model, but also to the randomness of human behavior and the interaction of countless minor influences that are not. It is understood that the introduction of a random disturbance into a deterministic model is not intended merely to paper over its inadequacies. It is essential to examine the results of the study, in an *ex post* analysis, to ensure that the allegedly random, unexplained factor is truly unexplainable. If it is not, the model is, in fact, inadequate.<sup>3</sup> The stochastic element endows the model with its statistical properties. Observations on the variable(s) under study are thus taken to be the outcomes of a random process. With a sufficiently detailed stochastic structure and adequate data, the analysis will become a matter of deducing the properties of a probability distribution. The tools and methods of mathematical statistics will provide the operating principles.

A model (or theory) can never truly be confirmed unless it is made so broad as to include every possibility. But it may be subjected to ever more rigorous scrutiny and, in the face of contradictory evidence, refuted. A deterministic theory will be invalidated by a single contradictory observation. The introduction of stochastic elements into the model changes it from an exact statement to a probabilistic description about expected outcomes and carries with it an important implication. Only a preponderance of contradictory evidence can convincingly invalidate the probabilistic model, and what constitutes a preponderance of evidence is a matter of interpretation. Thus, the probabilistic model is less precise but at the same time, more robust.<sup>4</sup>

The process of econometric analysis departs from the specification of a theoretical relationship. We initially proceed on the optimistic assumption that we can obtain precise measurements on all the variables in a correctly specified model. If the ideal conditions are met at every step, the subsequent analysis will be routine. Unfortunately, they rarely are. Some of the difficulties one can expect to encounter are the following:

- The data may be badly measured or may correspond only vaguely to the variables in the model. "The interest rate" is one example.

<sup>3</sup> In the example given earlier, the estimated constant term in the linear least squares regression is negative. Is the theory wrong, or is the finding due to random fluctuation in the data? Another possibility is that the theory is broadly correct, but the world changed between 1936 when Keynes devised his theory and 2000–2009 when the data (outcomes) were generated. Or, perhaps linear least squares is not the appropriate technique to use for this model, and that is responsible for the inconvenient result (the negative intercept).

<sup>4</sup> See Keuzenkamp and Magnus (1995) for a lengthy symposium on testing in econometrics.

- Some of the variables may be inherently unmeasurable. “Expectations” is a case in point.
- The theory may make only a rough guess as to the correct form of the model, if it makes any at all, and we may be forced to choose from an embarrassingly long menu of possibilities.
- The assumed stochastic properties of the random terms in the model may be demonstrably violated, which may call into question the methods of estimation and inference procedures we have used.
- Some relevant variables may be missing from the model.
- The conditions under which data are collected lead to a sample of observations that is systematically unrepresentative of the population we wish to study.

The ensuing steps of the analysis consist of coping with these problems and attempting to extract whatever information is likely to be present in such obviously imperfect data. The methodology is that of mathematical statistics and economic theory. The product is an econometric model.

## 1.6 PLAN OF THE BOOK

Our objective in this survey is to develop in detail a set of tools, then use those tools in applications. The following set of applications will include many that readers will use in practice. But it is not exhaustive. We will attempt to present our results in sufficient generality that the tools we develop here can be extended to other kinds of situations and applications not described here.

One possible approach is to organize (and orient) the areas of study by the type of data being analyzed—cross section, panel, discrete data, then time series being the obvious organization.

Alternatively, we could distinguish at the outset between micro- and macroeconometrics.<sup>5</sup> Ultimately, all of these will require a common set of tools, including, for example, the multiple regression model, the use of moment conditions for estimation, instrumental variables (IV), and maximum likelihood estimation. With that in mind, the organization of this book is as follows: The first half of the text develops fundamental results that are common to all the applications. The concept of multiple regression and the linear regression model in particular constitutes the underlying platform of most modeling, even if the linear model itself is not ultimately used as the empirical specification. This part of the text concludes with developments of IV estimation and the general topic of panel data modeling. The latter pulls together many features of modern econometrics, such as, again, IV estimation, modeling heterogeneity, and a rich variety of extensions of the linear model. The second half of the text presents a variety

---

<sup>5</sup> An excellent reference on the former that is at a more advanced level than this text is Cameron and Trivedi (2005). There does not appear to be available a counterpart, large-scale pedagogical survey of macroeconometrics that includes both econometric theory and applications. The numerous more focused studies include books such as Bardsen et al. (2005).

of topics. Part III is an overview of estimation methods. Finally, Parts IV and V present results from microeconometrics and macroeconometrics, respectively. The broad outline is as follows:

### **I. Regression Modeling**

Chapters 2 through 6 present the multiple linear regression model. We will discuss specification, estimation, and statistical inference. This part develops the ideas of estimation, robust analysis, functional form, and principles of model specification.

### **II. Generalized Regression, Instrumental Variables, and Panel Data**

Chapter 7 extends the regression model to nonlinear functional forms. The method of instrumental variables is presented in Chapter 8. Chapters 9 and 10 introduce the generalized regression model and systems of regression models. This section ends with Chapter 11 on panel data methods.

### **III. Estimation Methods**

Chapters 12 through 16 present general results on different methods of estimation including GMM, maximum likelihood, and simulation-based methods. Various estimation frameworks, including non- and semiparametric and Bayesian estimation, are presented in Chapters 12 and 16.

### **IV. Microeconomic Methods**

Chapters 17 through 19 are about microeconometrics, discrete choice modeling, limited dependent variables, and the analysis of data on events—how many occur in a given setting and when they occur. Chapters 17 through 19 are devoted to methods more suited to cross sections and panel data sets.

### **V. Macroeconomic Methods**

Chapters 20 and 21 focus on time-series modeling and macroeconometrics.

### **VI. Background Materials**

Appendices A through E present background material on tools used in econometrics including matrix algebra, probability and distribution theory, estimation, and asymptotic distribution theory. Appendix E presents results on computation. The data sets used in the numerical examples are described in Appendix F. The actual data sets and other supplementary materials can be downloaded from the author's Web site for the text: <http://people.stern.nyu.edu/wgreene/Text/>.

## **1.7 PRELIMINARIES**

Before beginning, we note some specific aspects of the presentation in the text.

### **1.7.1 NUMERICAL EXAMPLES**

There are many numerical examples given throughout the discussion. Most of these are either self-contained exercises or extracts from published studies. In general, their purpose is to provide a limited application to illustrate a method or model. The reader can replicate them with the data sets provided. This will generally not entail attempting to replicate the full published study. Rather, we use the data sets to provide applications that relate to the published study in a limited fashion that also focuses on a particular